Study Guide #1 (254)

1. $\exists t$: all reals. $\frac{1}{1+\cos t} \Rightarrow 1+\cos t \neq 0 \Rightarrow \cos t \neq -1 \Rightarrow t \neq \pi$ or $(2n+1)\pi$ odd multiples

$\sqrt{t-1}$: $t-1 \geq 0 \Rightarrow t \geq 1$

$\therefore$ Domain of the function $\{t \mid t \geq 1$ and $t \neq (2k+1)\pi,\ k \in \{0,1,2..\}\}$

the function is not continuous since the j coordinate has an asymptote at all odd multples of π. None of these are removable.

2. $\Delta x = -1-(-2) = -1+2 = 1$

$\Delta y = 4-5 = -1$

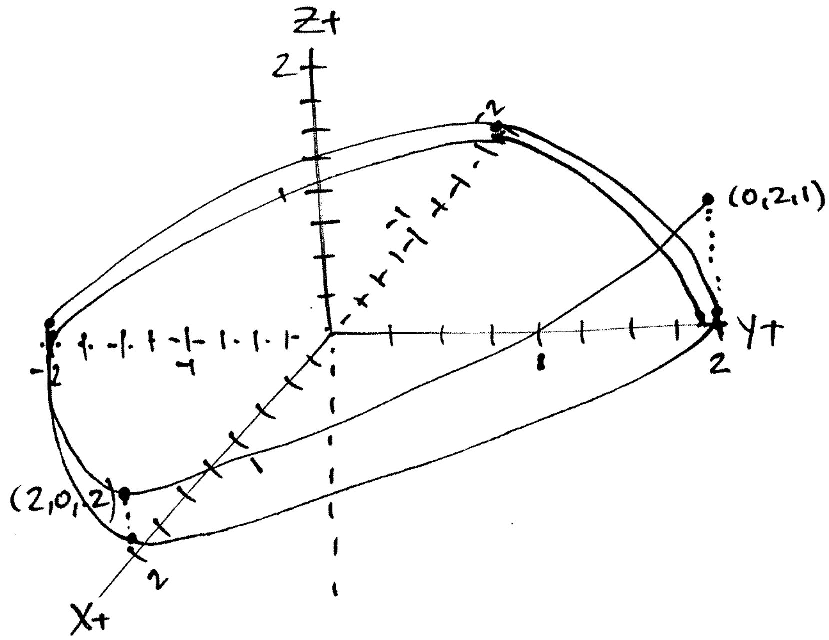
$\Delta z = 9-(-3) = 9+3 = 12$

$\vec{r}(t) = (\Delta x t + x_0)\hat{\imath} + (\Delta y t + y_0)\hat{\jmath} + (\Delta z t + z_0)\hat{k} \quad t \in [0,1]$

$= (t-2)\hat{\imath} + (5-t)\hat{\jmath} + (12t-3)\hat{k}$

3. note: the sine and cosine pair indicate the projection in the x-y plane is a circle, and the exponential in the z direction indicates a "helix"-like shape with exponentially decreasing height to the plane. Plot 2 loops. Negative angles will give you larger exponential values.

t	x	y	z
0	0	2	1
$\pi/2$	2	0	$\approx .208$
π	0	-2	$\approx .04$
$3\pi/2$	-2	0	$\approx .009$
2π	0	2	$\approx .002$
$5\pi/2$	2	0	$\approx .0003$
3π	0	-2	$\approx .00008$
$7\pi/2$	-2	0	≈ 0
4π	0	0	≈ 0



4. $x^2+y^2+z^2=4 \qquad z=2-x = 2-1-\sin t = 1-\sin t$

$x^2+y^2+(2-x)^2=4$

$x^2+y^2+\cancel{4}-2x+x^2=\cancel{4}$

$2x^2-2x+y^2=0 \qquad x^2=(1+\sin t)^2$

$\cancel{2}+4\sin t+2\sin^2 t-\cancel{2}-2\sin t+y^2=0 \qquad 1+2\sin t+\sin^2 t$

$-2\sin t \mp 2\sin^2 t = y^2$

$-2(\sin t)(1+\sin t)=y^2 \Rightarrow y=\sqrt{-2\sin t\,(1+\sin t)}$

this will only be defined when $\sin t \le 0 \quad t\in[-\pi,0]$

$\vec{r}(t)=$

$(1+\sin t)\hat{i}+\left(\sqrt{-2\sin t\,(1+\sin t)}\right)\hat{j}+(1-\sin t)\hat{k}$

5. $r(t)=t\vec{i}+3t\vec{j}+t^2\vec{k} \qquad u(t)=4t\hat{i}+t^2\hat{j}+t^3\hat{k}$

a) $\vec{r}'(t)=\hat{i}+3\hat{j}+2t\hat{k}$

$\vec{r}''(t)= \qquad 2\hat{k}$

b) $r(t)\times u(t) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ t & 3t & t^2 \\ 4t & t^2 & t^3 \end{vmatrix} = (3t^4-t^4)\hat{i}-(t^4-4t^3)\hat{j}+(t^3-12t^2)\hat{k}$

$= 2t^4\hat{i}+(4t^3-t^4)\hat{j}+(t^3-12t^2)\hat{k}$

$D_t[r(t)\times u(t)] = 8t^3\hat{i}+(12t^2-4t^3)\hat{j}+(3t^2-24t)\hat{k}$

or use the chain rule

$u'(t)=4\hat{i}+2t\hat{j}+3t^2\hat{k}$

$D_t[r(t)\times u(t)] = r'(t)\times u(t)+r(t)\times u'(t) =$

$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 3 & 2t \\ 4t & t^2 & t^3 \end{vmatrix} + \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ t & 3t & t^2 \\ 4 & 2t & 3t^2 \end{vmatrix} =$

$$(3t^3-2t^3)\hat{\imath}-(t^3-8t^2)\hat{\jmath}+(t^2-12t)\hat{k}+(9t^3-2t^3)\hat{\imath}-(3t^3-4t^2)\hat{\jmath}$$
$$+(2t^2-12t)\hat{k}=$$

$$(t^3+7t^3)\hat{\imath}-(t^3-8t^2+3t^3-4t^2)\hat{\jmath}+(t^2-12t+2t^2-12t)\hat{k}=$$

$$8t^3\hat{\imath}+(12t^2-4t^3)\hat{\imath}+(3t^2-24t)\hat{k}$$

c) $\|\vec{r}\,'(t)\| = \sqrt{(1)^2+(3)^2+(2t)^2} = \sqrt{10+4t^2}$

$D_t\|\vec{r}\,'(t)\| = \frac{1}{2}(10+4t^2)^{-1/2}\,(8t) = \dfrac{4t}{\sqrt{10+4t^2}}$

d) $\int \vec{r}(t)\,dt = \int t\hat{\imath}+3t\hat{\jmath}+t^2\hat{k}\;dt =$

$$\int t\,dt\,\hat{\imath}+\int 3t\,dt\,\hat{\jmath}+\int t^2dt\,\hat{k}$$
$$\left(\tfrac{1}{2}t^2+C_1\right)\hat{\imath}+\left(\tfrac{3}{2}t^2+C_2\right)\hat{\jmath}+\left(\tfrac{1}{3}t^3+C_3\right)\hat{k}$$

e) $\int_{-1}^{1}\|r'(t)\|\,dt = \int_{-1}^{1}\sqrt{10+4t^2}\;dt$

$2t=\sqrt{10}\tan\theta$

$dt=\frac{\sqrt{10}}{2}\sec^2\theta\,d\theta$

$\sqrt{10+4t^2}=\sqrt{10}\sec\theta$

$\int\frac{\sqrt{10}}{2}\sec^2\theta\,\sqrt{10}\sec\theta\,d\theta =$

$5\int\sec^3\theta\,d\theta$

$u=\sec\theta \quad dv=\sec^2\theta\,d\theta$

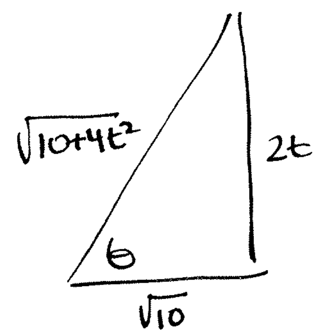
$\frac{du}{d\theta}=\sec\theta\tan\theta \quad v=\tan\theta$

$\int\sec^3\theta\,d\theta = \sec\theta\tan\theta-\int\sec\theta\tan^2\theta\,d\theta$

$\int\sec^3\theta\,d\theta = \sec\theta\tan\theta-\int\sec\theta(\sec^2\theta-1)\,d\theta \Rightarrow$

$\int\sec^3\theta\,d\theta = \sec\theta\tan\theta-\int\sec^3\theta\,d\theta+\int\sec\theta\,d\theta$

$\frac{2\int\sec^3\theta\,d\theta}{2} = \frac{\sec\theta\tan\theta+\ln|\sec\theta+\tan\theta|+C}{2}$



$5\int \sec^3\theta\, d\theta = \frac{5}{2}\sec\theta\tan\theta + \frac{5}{2}\ln|\sec\theta + \tan\theta| + C$ (4)

$\sec\theta = \frac{\sqrt{10+4t^2}}{\sqrt{10}}$

$\tan\theta = \frac{2t}{\sqrt{10}}$

$$\int_{-1}^{1}\sqrt{10+4t^2}\,dt = \frac{5}{2}\sqrt{\frac{10+4t^2}{10}}\cdot\frac{2t}{\sqrt{10}} + \frac{5}{2}\ln\left|\sqrt{\frac{10+4t^2}{10}} + \frac{2t}{\sqrt{10}}\right|\Bigg]_{-1}^{1}$$

$$\frac{5}{2}\sqrt{\frac{10+4}{10}}\cdot\frac{2(1)}{\sqrt{10}} + \frac{5}{2}\ln\left|\sqrt{\frac{10+4}{10}} + \frac{2(1)}{\sqrt{10}}\right| - \frac{5}{2}\sqrt{\frac{10+4}{10}}\cdot\frac{2(-1)}{\sqrt{10}} - \frac{5}{2}\ln\left|\sqrt{\frac{10+4}{10}} + \frac{2(-1)}{\sqrt{10}}\right|$$

$$= \frac{5}{\cancel{2}}\sqrt{\frac{16}{100}}\cdot\cancel{2} + \frac{5}{2}\ln\left|\sqrt{\frac{16}{10}} + \frac{2}{\sqrt{10}}\right| + \frac{5}{\cancel{2}}\sqrt{\frac{16}{100}}\cdot\cancel{2} - \frac{5}{2}\ln\left|\sqrt{\frac{16}{10}} - \frac{2}{\sqrt{10}}\right|$$

$$\cancel{5}\cdot\frac{4}{\cancel{10}\,2} + \frac{5}{2}\ln\left|\frac{4+2}{\sqrt{10}}\right| + 5\,\frac{4}{2\,10} - \frac{5}{2}\ln\left|\frac{4-2}{\sqrt{10}}\right|$$

$$2 + \frac{5}{2}\ln\left|\frac{6}{\sqrt{10}}\right| + 2 - \frac{5}{2}\ln\left|\frac{2}{\sqrt{10}}\right|$$

$$4 + \frac{5}{2}\left[\ln\left|\frac{6}{\sqrt{10}}\right| - \ln\left|\frac{2}{\sqrt{10}}\right|\right] = 4 + \frac{5}{2}\ln\left[\frac{6/\cancel{\sqrt{10}}}{2/\cancel{\sqrt{10}}}\right] =$$

$$\boxed{4 + \frac{5}{2}\ln 3}$$

6. $\vec{a}(t) = -\cos t\,\hat{i} - \sin t\,\hat{j}$ $\quad v(0) = \vec{j} + \vec{k}$ $\quad r(0) = \vec{i}$

$\int \vec{a}(t)\,dt = v(t)$

$= \int -\cos t\,dt\,\hat{i} - \int \sin t\,dt\,\hat{j} + \int 0\,dt\,\hat{k}$

$(-\sin t + C_1)\hat{i} + (\cos t + C_2)\hat{j} + C_3\hat{k} \Leftarrow$ Pay attention to this constant!

$v(0) \Rightarrow$

$-\sin(0) + C_1 = 0 \Rightarrow C_1 = 0$

$\cos(0) + C_2 = 1 \Rightarrow C_2 = 0$

$C_3 = 1 \Rightarrow C_3 = 1$

$v(t) = (-\sin t)\hat{i} + \cos t\hat{j} + \hat{k}$

$\int v(t)\,dt = r(t)$

$\int -\sin t\,dt\,\hat{i} + \int \cos t\,dt\,\hat{j} + \int 1\,dt\,\hat{k}$

$(\cos t + C_1)\hat{i} + (\sin t + C_2)\hat{j} + (t + C_3)\hat{k}$

$r(0) = \hat{i}$

$\cos(0) + C_1 = 1 \Rightarrow C_1 = 0$

$\sin(0) + C_2 = 0 \Rightarrow C_2 = 0$

$(0) + C_3 = 0 \Rightarrow C_3 = 0$

$r(t) = \cos t\,\hat{i} + \sin t\hat{j} + t\hat{k}$

$r(2) = \cos 2\,\hat{i} + \sin(2)\hat{j} + 2\hat{k}$ t is in radians so it doesn't reduce nicely.

7. $\vec{r}(t) = 2\sin t\,\hat{i} + 2\cos t\,\hat{j}$

a) $\vec{r}\,'(t) = 2\cos t\,\hat{i} - 2\sin t\,\hat{j}$

$\|\vec{r}\,'(t)\| = \sqrt{4\cos^2 t + 4\sin^2 t} = \sqrt{4} = 2$

$\vec{T}(t) = \frac{r'(t)}{\|r'(t)\|} = \cos t\,\hat{i} - \sin t\,\hat{j}$

$\vec{T}\,'(t) = -\sin t\hat{i} - \cos t\,\hat{j}$

$\|\vec{T}\,'(t)\| = 1$

$\frac{T'(t)}{\|T'(t)\|} = \vec{N}(t) = -\sin t\,\hat{i} - \cos t\,\hat{j}$

b) $\vec{r}(t) = \cos 3t\,\hat{i} + 2\sin 3t\,\hat{j} + \hat{k}$

$\vec{r}\,'(t) = -3\sin 3t\,\hat{i} + 6\cos 3t\,\hat{j}$

$\|\vec{r}\,'(t)\| = \sqrt{9\sin^2 3t + 36\cos^2 3t} = \sqrt{9(\sin^2 3t + 4\cos^2 3t)} = 3\sqrt{\sin^2 3t + \cos^2 3t + 3\cos^2 3t}$

$= 3\sqrt{1 + 3\cos^2 3t}$

$$\vec{T}(t) = \frac{\vec{r}\,'(t)}{\|\vec{r}\,'(t)\|} = \frac{-\cancel{3}\sin 3t\,\hat{\imath} + \overset{2}{\cancel{6}}\cos 3t\,\hat{\jmath}}{\cancel{3}\sqrt{1+3\cos^2 3t}} =$$

$$\frac{1}{\sqrt{1+3\cos^2 3t}}\left[-\sin 3t\,\hat{\imath} + 2\cos 3t\,\hat{\jmath}\right]$$

$$\vec{T}\,'(t) = -\frac{1}{\cancel{2}}(1+3\cos^2 3t)^{-3/2}\,3\cdot\cancel{2}\cos 3t\sin 3t\cdot 3\left[-\sin 3t\,\hat{\imath} + 2\cos 3t\,\hat{\jmath}\right]$$

$$+(1+3\cos^2 3t)^{-1/2}\left[-3\cos 3t\,\hat{\imath} - 6\sin 3t\,\hat{\jmath}\right]$$

$$\frac{-9\cos 3t\sin 3t}{(1+3\cos^2 3t)^{3/2}}\left[-\sin 3t\,\hat{\imath} + 2\cos 3t\,\hat{\jmath}\right] + \frac{1}{(1+3\cos^2 3t)^{1/2}}\left[-3\cos 3t\,\hat{\imath} - 6\sin 3t\,\hat{\jmath}\right]$$

$$\frac{-9\cos 3t\sin 3t(-\sin 3t)\hat{\imath} - 9\cos 3t\sin 3t\cdot 2\cos 3t\,\hat{\jmath} - (1+3\cos^2 3t)3\cos 3t\,\hat{\imath} - 6\sin 3t(1+3\cos^2 3t)\hat{\jmath}}{(1+3\cos^2 3t)^{3/2}}$$

$$\frac{\left[+9\cos 3t\overset{(1-\cos^2 3t)}{\sin^2 3t} - 3\cos 3t - 9\cos^3 3t\right]\hat{\imath} + \left[-18\cos^2 3t\sin 3t - 6\sin 3t - 18\cos^2 3t\sin 3t\right]\hat{\jmath}}{(1+3\cos^2 3t)^{3/2}}$$

$$\frac{\left[9\cos 3t - 9\cos^3 3t - 3\cos 3t - 9\cos^3 3t\right]\hat{\imath} + \left[\overset{-36\sin 3t\,+\,36\sin^3 3t}{-36(1-\sin^2 3t)\sin 3t} - 6\sin 3t\right]\hat{\jmath}}{(1+3\cos^2 3t)^{3/2}}$$

$$\frac{\left[6\cos 3t - 18\cos^3 3t\right]\hat{\imath} + \left[36\sin^3 3t - 42\sin 3t\right]\hat{\jmath}}{(1+3\cos^2 3t)^{3/2}}$$

$$\|\vec{T}\,'(t)\| = \sqrt{\frac{(6\cos 3t - 18\cos^3 3t)^2 + (36\sin^3 3t - 42\sin 3t)^2}{(1+3\cos^2 3t)^3}}$$

$$\sqrt{\frac{36\cos^2 3t - 216\cos^4 3t + 324\cos^6 3t + 1296\sin^6 3t - 3024\sin^4 3t + 1764\sin^2 3t}{\cdot (1+3\cos^2 3t)^3}}$$

$$= \frac{3\cdot 2\sqrt{\underset{41}{\cancel{36}} + \underset{48}{\cancel{1728}}\sin^2 3t - \underset{6}{\cancel{216}}\cos^4 3t + \underset{9}{\cancel{324}}\cos^6 3t + \underset{36}{\cancel{1296}}\sin^6 3t - \underset{84}{\cancel{3024}}\sin^4 3t}}{(1+3\cos^2 3t)^{3/2}}$$

$$\vec{N}(t) = \frac{\vec{T}'(t)}{\|\vec{T}'(t)\|} = \frac{[\cancel{6}\cos 3t - \overset{3}{\cancel{18}}\cos^3 3t]\hat{i} + [\overset{6}{\cancel{36}}\sin^3 3t - \overset{7}{\cancel{42}}\sin 3t]\hat{j}}{\cancel{(1+3\cos^2 3t)^{3/2}}} \cdot \frac{\cancel{(1+3\cos^2 3t)^{3/2}}}{\cancel{6}\sqrt{\quad}}$$

$$= \frac{[\cos 3t - 3\cos^3 3t]\hat{i} + [6\sin^3 3t - 7\sin 3t]\hat{j}}{\sqrt{1 + 48\sin^2 3t - 6\cos^4 3t + 9\cos^6 3t + 36\sin^6 3t - 84\sin^4 3t}}$$

this can probabably be reduced by converting everything to powers of sine, but there is no need since we don't need to go further.

8. $\vec{r}(t) = -t\hat{i} + 4t\hat{j} + 3t\vec{k}$

$\vec{r}'(t) = -1\hat{i} + 4\hat{j} + 3\hat{k}$

$\|r'(t)\| = \sqrt{1^2 + 4^2 + 3^2} = \sqrt{1+16+9} = \sqrt{26}$

$S = \int_0^1 \sqrt{26}\,dt = \sqrt{26}$

$u(t) = 4t\hat{i} - \cos t\,\hat{j} + \sin t\,\hat{k}$

$u'(t) = 4\hat{i} + \sin t\,\hat{j} + \cos t\,\hat{k}$

$\|u'(t)\| = \sqrt{4^2 + \sin^2 t + \cos^2 t} = \sqrt{16+1} = \sqrt{17}$ $\quad \int_0^1 \sqrt{17}\,dt = \sqrt{17}$

$s_r = \int_0^t \sqrt{26}\, dt = \sqrt{26}\, t \qquad t = \frac{s}{\sqrt{26}}$

$s_u = \int_0^t \sqrt{17}\, dt = \sqrt{17}\, t \qquad t = \frac{s}{\sqrt{17}}$

9. a) $\vec{r}(t) = t\hat{i} + \sin t\,\hat{j}$

$K = \frac{\|r'(t) \times r''(t)\|}{\|r'(t)\|^3}$

$\vec{r}\,'(t) = 1\hat{i} + \cos t\,\hat{j}$

$\vec{r}\,''(t) = \quad -\sin t\,\hat{j}$

$\|\vec{r}\,'(t)\| = \sqrt{1 + \cos^2 t}$

$\vec{r}\,'(t) \times \vec{r}\,''(t) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & \cos t & 0 \\ 0 & -\sin t & 0 \end{vmatrix} = (0)\hat{i} - 0\hat{j} + (-\sin t - 0)\hat{k}$

$= -\sin t\,\hat{k}$

$\|r'(t) \times r''(t)\| = \sqrt{\sin^2 t} = \sin t$

$K = \frac{\sin t}{\left(\sqrt{1+\cos^2 t}\right)^3}$

b) $\vec{r}(t) = 5\cos t\,\hat{i} + 4\sin t\,\hat{j}$

$\|\vec{r}\,'(t)\| = \sqrt{25\sin^2 t + 16\cos^2 t} =$

$\sqrt{16\sin^2 t + 9\sin^2 t + 16\cos^2 t}$

$\sqrt{16 + 9\sin^2 t}$

$\vec{r}\,'(t) = -5\sin t\,\hat{i} + 4\cos t\,\hat{j}$

$\vec{r}\,''(t) = -5\cos t\,\hat{i} - 4\sin t\,\hat{j}$

$\vec{r}\,'(t) \times \vec{r}\,''(t) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -5\sin t & 4\cos t & 0 \\ -5\cos t & -4\sin t & 0 \end{vmatrix} = (0)\hat{i} - (0)\hat{j} + (20\sin^2 t + 20\cos^2 t)\hat{k}$

$= 20\hat{k}$

$\|\vec{r}\,'(t) \times \vec{r}\,''(t)\| = 20$

$K = \frac{20}{\left(\sqrt{16 + 9\sin^2 t}\right)^3}$

$\vec{r}(t) =$

c) $2\cos 3t\,\hat{i} + 2\sin 3t\,\hat{j} + e^t\,\hat{k}$ (12)

$\vec{r}'(t) = -6\sin 3t\,\hat{i} + 6\cos 3t\,\hat{j} + e^t\,\hat{k}$

$\vec{r}''(t) = -18\cos 3t\,\hat{i} - 18\sin 3t\,\hat{j} + e^t\,\hat{k}$

$$\vec{r}'(t) \times \vec{r}''(t) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -6\sin 3t & 6\cos 3t & e^t \\ -18\cos 3t & -18\sin 3t & e^t \end{vmatrix} =$$

$$(6e^t\cos 3t + 18e^t\sin 3t)\hat{i} + (6e^t\sin 3t - 18e^t\cos 3t)\hat{j} + 108\hat{k}$$

$$6e^t(\cos 3t + 3\sin 3t)\hat{i} + 6e^t(\sin 3t - 3\cos 3t)\hat{j} + 108\hat{k}$$

$$\|\vec{r}'(t) \times \vec{r}''(t)\| = \sqrt{36e^{2t}(\cos^2 3t + 6\cos 3t\sin 3t + 9\sin^2 3t + 36e^{2t}(\sin^2 3t - 6\cos 3t\sin 3t + 9\cos^2 3t) + 108^2}$$

$$= \sqrt{36e^{2t}(\underbrace{\cancel{\sin^2 3t + \cos^2 3t}}_{1}) + 36e^{2t}(\underbrace{\cancel{6\cos 3t\sin 3t - 6\cos 3t\sin 3t}}_{0}) + 36e^{2t}(\underbrace{\cancel{9\sin^2 3t + 9\cos^2 3t}}_{9}) + 108^2}$$

$$= \sqrt{36e^{2t} + 324e^{2t} + 108^2} =$$

$$\sqrt{360e^{2t} + 11664} = \sqrt{36(10e^{2t} + 324)} =$$

$$6\sqrt{10e^{2t} + 324}$$

$$\|\vec{r}'(t)\| = \sqrt{36\sin^2 3t + 36\cos^2 3t + e^{2t}} = \sqrt{36 + e^{2t}}$$

$$K = \frac{6\sqrt{10e^{2t} + 324}}{(\sqrt{36 + e^{2t}})^3}$$

10. a) $y = 2x + \frac{4}{x}$ $\qquad K = \frac{|y''|}{[1+(y')^2]^{3/2}}$

$y' = 2 - \frac{4}{x^2}$ $\qquad (y')^2 = 4 - \frac{8}{x^2} + \frac{16}{x^4}$

$y'' = \frac{8}{x^3}$ $\qquad 1+(y')^2 = 5 - \frac{8}{x^2} + \frac{16}{x^4}$

$$K = \frac{8/x^3}{\left(5 - \frac{8}{x^2} + \frac{16}{x^4}\right)^{3/2}}$$

b) $y = \cos 2x$ $\qquad K = \frac{|4\cos 2x|}{[1 + 4\sin^2 2x]^{3/2}}$

$y' = -2\sin 2x$

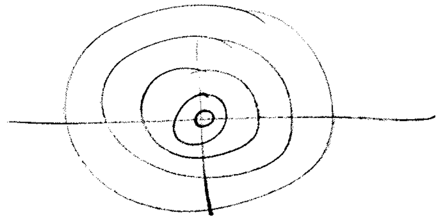
$y'' = -4\cos 2x$

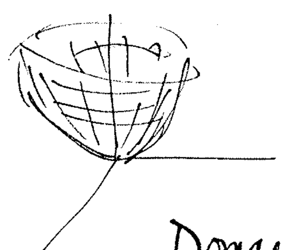
c) $y = x^{2/3}$ $\qquad K = \frac{\frac{2}{9}x^{-4/3}}{[1 + \frac{4}{9}x^{-2/3}]^{3/2}}$

$y' = \frac{2}{3}x^{-1/3}$

$y'' = -\frac{2}{9}x^{-4/3}$

11. a) $f(x,y) = x^2 + y^2$ $\qquad$ paraboloid





Domain: $\{(x,y) \mid x,y \in \mathbb{R}\}$

Range: $z \geq 0$

b) $z = \frac{x+y}{xy}$ $\qquad$ Domain: $\{(x,y) \mid x \neq 0, y \neq 0\}$

Range: $z \neq 0$

$$zxy = x + y$$
$$zxy - y = x$$
$$y(zx - 1) = x$$
$$y = \frac{x}{zx - 1}$$

for $z = 0 \rightarrow y = -x$

$z = 1 \rightarrow y = \frac{x}{x-1}$

$z = 2 \rightarrow y = \frac{x}{2x-1}$

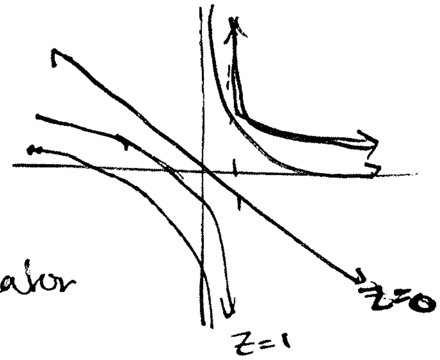
etc.

you can graph these in your calculator

also try

$z = 1/2$, $z = 1/3$, etc

$z = -1, -2$, etc.



this graph is a hyperboloid turned on its side

c) $f(x,y) = \ln(4 - x - y)$

$$z = \ln(4 - x - y)$$
$$e^z = 4 - x - y$$
$$y = 4 - x - e^z$$

$z = 0 \rightarrow y = 4 - x - e^0 = 3 - x$

$z = 1 \rightarrow y = (4 - e^2) - x$

$z = 2 \rightarrow y = (4 - e^3) - x$

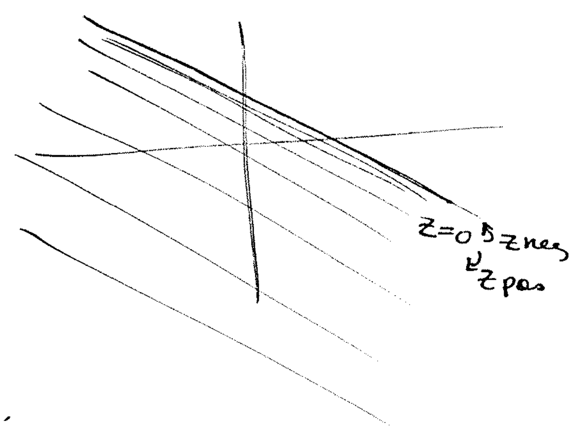
also try $z = -1, -2, -3$, etc.

$z = -1 \rightarrow y = (4 - \frac{1}{e}) - x$

$z = -2 \rightarrow y = (4 - \frac{1}{e^2}) - x$

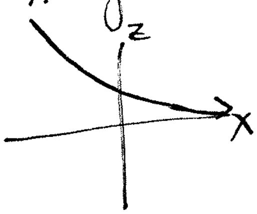
intercepts exponentially approaching 4

The graph look like the sheet of a log function with its asymptote turned to lie along the line $y = -x + 4$ in the plane.



d) $f(x,y) = e^{-x}$

this graph has no y's so it's like a 2D graph

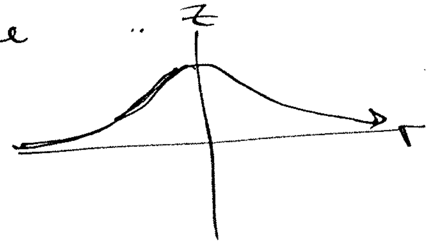


with y-coming out of the page

it's a sheet in this form

e) $f(x,y) = \dfrac{8}{1+x^2+y^2}$ this graph is a little easier to picture in polar

$f(r,\theta) = \dfrac{8}{1+r^2}$ in 2D this looks like



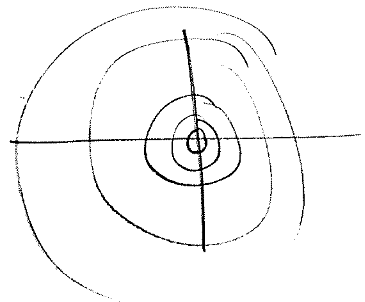
now revolve this around the z-axis.

$z = \dfrac{8}{1+x^2+y^2} \Rightarrow \cancel{z}(1+x^2+y^2) = \dfrac{8}{z} - 1$

$-1 \qquad -1$

$x^2+y^2 = \dfrac{8}{z} - 1$ this is a circle in the plane

The $\sqrt{\ }$ of this figure is the radius



$z=8$ point

$z=4 \Rightarrow x^2+y^2 = \frac{8}{4} - 1 = 2-1 \Rightarrow x^2+y^2 = 1$

circle of radius one

$z=1$

$x^2+y^2 = 7$

etc. not defined at $z=0$

graph approaches the plane asymptotically

12. a) $\lim\limits_{(x,y)\to(0,0)} \dfrac{x-y}{\sqrt{x-y}} \cdot \dfrac{\sqrt{x-y}}{\sqrt{x-y}} = \dfrac{\cancel{(x-y)}\sqrt{x-y}}{\cancel{x-y}} = \sqrt{x-y}$ rationalize

$\lim\limits_{(x,y)\to(0,0)} \sqrt{x-y} = 0$

b) $\lim\limits_{(x,y)\to(0,0)} \dfrac{-xy^2}{x^2+y^2}$ try converting to polar $\lim\limits_{(r,\theta)\to(0,\theta)} \dfrac{-r\cos\theta\, \cancel{r^2}\sin^2\theta}{\cancel{r^2}} =$

$\lim\limits_{(r,\theta)\to(0,\theta)} -r\cos\theta\sin^2\theta = 0$

r is left

c) $\lim_{(x,y,z)\to(0,0,0)} \dfrac{z}{x^2+y^2-4} = \dfrac{0}{-4} = 0$

d) $\lim_{(x,y,z)\to(0,0,0)} \dfrac{1}{\sqrt{x^2+y^2+z^2}}$ Convert to spherical

$\lim_{(\rho,\varphi,\theta)\to(0,\varphi,\theta)} = \dfrac{1}{\rho} = \dfrac{1}{0} = \infty$ ρ is assumed to be positive

13. a) $f(x,y) = x^2y^3$

$f_x = 2xy^3$ $f_{xx} = 2y^3$ $f_{xy} = 6xy^2$ $f_{xyy} = 12xy$

$f_y = 3x^2y^2$

b) $f(x,y) = \ln\left(\frac{x}{y}\right) = \ln x - \ln y$

$f_x = \frac{1}{x}$ $f_{xx} = -\frac{1}{x^2}$ $f_{xy} = 0$ $f_{xyy} = 0$

$f_y = \frac{1}{y}$

c) $f(x,y) = \tan(2x-y)$

$f_x = \sec^2(2x-y)\cdot 2$ $f_{xx} = 8\sec^2(2x-y)\tan(2x-y)$

$f_y = \sec^2(2x-y)(-1)$

$f_{xy} = -4\sec^2(2x-y)\tan(2x-y)$

$f_{xyy} = -4\left[2\sec^2(2x-y)\tan^2(2x-y)(-1) + \sec^2(2x-y)\sec^2(2x-y)(-1)\right]$

$= 8\sec^2(2x-y)\tan^2(2x-y) + 4\sec^4(2x-y)$

d) $f(x,y) = \sqrt{x^2+y^2} = (x^2+y^2)^{1/2}$

$f_x = \frac{1}{2}(x^2+y^2)^{-1/2}\cdot 2x = x(x^2+y^2)^{-1/2} = \dfrac{x}{\sqrt{x^2+y^2}}$

$f_y = y(x^2+y^2)^{-1/2} = \dfrac{y}{\sqrt{x^2+y^2}}$

$$f_{xx} = 1(x^2+y^2)^{-1/2} + x(-\tfrac{1}{\cancel{2}})(x^2+y^2)^{-3/2}\cdot(\cancel{2}x)$$

$$= (x^2+y^2)^{-1/2} - x^2(x^2+y^2)^{-3/2}$$

$$f_{xy} = -xy\,(x^2+y^2)^{-3/2}$$

$$f_{xyy} = -x\,(x^2+y^2)^{-3/2} - xy\,(-\tfrac{3}{\cancel{2}})(x^2+y^2)^{-5/2}(\cancel{2}y)$$

$$- x\,(x^2+y^2)^{-3/2} + 3xy^2(x^2+y^2)^{-5/2}$$

14. $f(x,y,z) = x^2 - 3xy + 4yz + z^3$

$$f_x = 2x - 3y$$
$$f_y = -3x + 4z$$
$$f_z = 4y + 3z^2$$

15. $w = 2z^3 y \sin x$

$w_x = 2z^3 y \cos x \qquad w_x(\pi,2,1) = 2(1)^3(2)(-1) = -4$

$w_y = 2z^3 \sin x \qquad w_y(\pi,2,1) = 0$

$w_z = 6z^2 y \sin x \qquad w_z(\pi,2,1) = 0$

$dw = w_x\,dx + w_y\,dy + w_z\,dz$

$= -4(-\pi/6) + 0(.1) + 0(.05)$

$= 4\pi/6 = \boxed{2\pi/3}$

$dx = 5\pi/6 - \pi = -\pi/6$

$dy = 2.1 - 2 = .1$

$dz = 1.05 - 1 = .05$