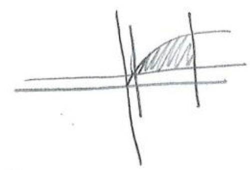
Name KEY

Math 254, Quiz #9, Winter 2012

Instructions: Show all work. Use exact answers unless directed otherwise (with the exception of some application problems). Problems with answers only will rarely receive full credit. Be sure to read each problem carefully and complete all parts.

1. Integrate. For each integral, sketch the graph of the region. (5 points each)

a. $\int_1^4 \int_1^{\sqrt{x}} 2ye^{-x}\,dydx$



$\int_1^4 y^2\Big]_1^{\sqrt{x}} e^{-x}\,dx$

$\int_1^4 xe^{-x} - e^{-x}\,dx$

$\int xe^{-x}\,dx$

$u = x \quad dv = e^{-x}dx$

$du = dx \quad v = -e^{-x}dx$

$\int_1^4 xe^{-x} - e^{-x}\,dx =$

$-xe^{-x}\Big|_1^4 + \cancel{\int_1^4 e^{-x}dx} - \cancel{\int_1^4 e^{-x}dx} = -4e^{-4} + 1e^{-1} = \boxed{\frac{1}{e} - \frac{4}{e^4}}$

b. $\int_0^{\pi/2} \int_0^{\sin\theta} \theta r\,drd\theta = \int_0^{\pi/2} \theta \, \frac{1}{2}r^2\Big]_0^{\sin\theta} d\theta =$

$\frac{1}{2}\int_0^{\pi/2} \theta \sin^2\theta \, d\theta = \frac{1}{2}\frac{1}{2}\int_0^{\pi/2} \theta - \theta\cos 2\theta \, d\theta$

$u = \theta \quad dv = \cos 2\theta\, d\theta$

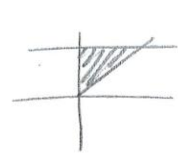
$du = d\theta \quad v = \frac{1}{2}\sin 2\theta\, d\theta$

$\frac{1}{4}\left[\frac{1}{2}\theta^2 - \frac{1}{2}\theta\sin 2\theta\Big|_0^{\pi/2} + \int_0^{\pi/2} \frac{1}{2}\sin 2\theta\, d\theta\right] =$

$\frac{1}{4}\left[\frac{1}{2}\theta^2 - \frac{1}{2}\theta\sin 2\theta - \frac{1}{2}\cdot\frac{1}{2}\cos 2\theta\right|_0^{\pi/2} = \frac{1}{4}\cdot\frac{1}{2}\left(\frac{\pi}{2}\right)^2 - \cancel{\frac{1}{4}\frac{1}{2}\frac{\pi}{2}\sin\pi} - \frac{1}{4}\frac{1}{4}\cos\pi - \cancel{\frac{1}{4}\cdot\frac{1}{2}(0)^2} + \cancel{\frac{1}{2}(0)\sin 0} + \frac{1}{4}\cdot\frac{1}{4}\cos 0 = \boxed{\frac{\pi^2}{32} + \frac{1}{8}}$

2. Find the volume bounded by the graphs $z = 1 - xy, y = x, y = 1$ in the first octant. (5 points)

$\to z \geq 0, x \geq 0, y \geq 0$



$\int_0^1 \int_0^y 1 - xy \, dx\,dy$

$\int_0^1 x - \frac{1}{2}x^2 y \Big|_0^y dy =$

$\int_0^1 y - \frac{1}{2}y^3\, dy = \frac{1}{2}y^2 - \frac{1}{8}y^4\Big|_0^1 = \frac{1}{2} - \frac{1}{8} - 0 = \boxed{\frac{3}{8}}$