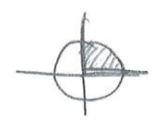
Name KEY

Math 254, Quiz #8, Winter 2012

Instructions: Show all work. Use exact answers unless directed otherwise (with the exception of some application problems). Problems with answers only will rarely receive full credit. Be sure to read each problem carefully and complete all parts.

1. Find the absolute extrema of the function $f(x,y) = x^2 + xy + \frac{1}{2}y^2 - 2x + y$ in the region bounded by $x \geq 0, y \geq 0, x^2 + y^2 \leq 1$. Sketch the region. (8 points)



$f_x = 2x + y - 2 = 0$

$f_y = x + y + 1 = 0$ subtract

$x \quad - 3 = 0 \qquad x = 3$

$3 + y + 1 = 0$

$y = -4 \qquad (3,4)$ not in region

$f(1,0) = 1 + 0 + 0 - 2(1) + 0 = \boxed{-1}$ MIN

other corners: $f(0,0) = 0$

$f(0,1) = 0 + 0 + \frac{1}{2} - 0 + 1 = \boxed{3/2}$ MAX

$x = 0 \Rightarrow f(0,y) = \frac{1}{2}y^2 + y \qquad f'(0,y) = y + 1 = 0 \Rightarrow y = -1$ not in region

$y = 0 \Rightarrow f(x,0) = x^2 - 2x \qquad f'(x,0) = 2x - 2 \Rightarrow x = 1 \qquad (1,0)$ also a corner

on circle: $f(x, \sqrt{1-x^2}) = x^2 + x\sqrt{1-x^2} + \frac{1}{2}(1-x^2) - 2x + \sqrt{1-x^2}$

$y = \sqrt{1-x^2} \qquad f'(x, \sqrt{1-x^2}) = 2x + \sqrt{1-x^2} + \frac{x \cdot (-2x)}{\sqrt{1-x^2}} + \frac{1}{2}(-2x) - 2 + \frac{-2x}{\sqrt{1-x^2}} = 0$

$\left[2x\sqrt{1-x^2} + (1-x^2) - 2x^2 - x\sqrt{1-x^2} - 2\sqrt{1-x^2}\right] / \sqrt{1-x^2} = [(x-2)\sqrt{1-x^2} + 1 - 3x^2]$ $\boxed{\text{never } 0}$

2. Find three positive integers x, y, and z whose product is 60 and the sum of squares is a minimum (7 points)

$xyz = 60 \Rightarrow z = \frac{60}{xy}$

$f(x,y,z) = x^2 + y^2 + z^2 \leftarrow$ min

$f(x,y) = x^2 + y^2 + 3600x^{-2}y^{-2}$

$f_x = 2x - 7200x^{-3}y^{-2} = 0 \Rightarrow 2x = \frac{7200}{x^3y^2} \Rightarrow x^4y^2 = 3600$

$f_y = 2y - 7200x^{-2}y^{-3} = 0 \Rightarrow 2y = \frac{7200}{x^2y^3} \Rightarrow y^4x^2 = 3600$

$x^4y^2 = y^4x^2 \Rightarrow x^4y^2 - y^4x^2 = 0 \qquad x^2y^2(x^2 - y^2) = 0$

$x = 0$ or $y = 0$ or $x^2 = y^2$

can't use zero since z would be undefined. $\quad x = y$ or $x = -y$

$\Rightarrow x^4y^2 = x^6 = 3600$

$\Rightarrow x^3 = \pm 60$

$x = \pm\sqrt[3]{60} \quad y = \pm\sqrt[3]{60} \qquad z = \frac{60}{\pm\sqrt[3]{60^2}} = \pm\sqrt[3]{60}$

$(x,y,z) = (\pm\sqrt[3]{60}, \pm\sqrt[3]{60}, \pm\sqrt[3]{60})$

for product to be positive use even