Name KEY

Math 254, Quiz #7, Winter 2012

Instructions: Show all work. Use exact answers unless directed otherwise (with the exception of some application problems). Problems with answers only will rarely receive full credit. Be sure to read each problem carefully and complete all parts.

1. Find the equation of the tangent plane to the graph $y - \ln xz^2 = 2$ at the point (e,2,1). Then find the symmetric equation of the normal line at the same point.(6 points)

$F(x,y,z) = y - \ln xz^2 - 2 = y - \ln x - 2\ln z - 2$

$\nabla F = \frac{-1}{x}\hat{\imath} + 1\hat{\jmath} - \frac{2}{z}\hat{k}$

$\nabla F(e,2,1) = \frac{-1}{e}\hat{\imath} + \hat{\jmath} - 2\hat{k}$

tangent plane: $-\frac{1}{e}(x-e) + 1(y-2) - 2(z-1) = 0$

normal line: $\frac{e-x}{e} = y-2 = \frac{1-z}{2}$

2. Find the critical point(s) of the function $f(x,y,z) = x^2 - 2y^2 + 3z^2 - xy + 4y + 9z + 16$. (4 points)

$f_x = 2x - y = 0 \Rightarrow y = 2x \qquad \Rightarrow y = \frac{8}{9}$

$f_y = -4y - x + 4 = 0 \Rightarrow -4(2x) - x + 4 = 0 \Rightarrow -9x + 4 = 0 \Rightarrow x = \frac{4}{9}$

$f_z = 6z + 9 = 0 \Rightarrow 6z = -9 \Rightarrow z = \frac{-3}{2}$

$(4/9,\ 8/9,\ -3/2)$

3. Find the critical point of the function $f(x,y) = -3x^2 - 2y^2 + 3x - 4y + 5$ and determine whether it is a max, min, saddle point or cannot be determined using the second partials test. (5 points)

$f_x = -6x + 3 = 0 \Rightarrow 6x = 3 \Rightarrow x = \frac{1}{2}$

$f_y = -4y - 4 = 0 \Rightarrow -4y = 4 \Rightarrow y = -1$

$f_{xx} = -6$

$f_{yy} = -4 \qquad (-6)(-4) - 0^2 = 24 > 0$

$f_{xy} = 0 \qquad f_{xx} < 0 \Rightarrow$ max