Name KEY

Math 254, Quiz #5, Winter 2012

Instructions: Show all work. Use exact answers unless directed otherwise (with the exception of some application problems). Problems with answers only will rarely receive full credit. Be sure to read each problem carefully and complete all parts.

1. Use the total differential to approximate the value of $\sin\left[1.05^2+0.95^2\right]-\sin\left(1^2+1^2\right)$. (7 points)

$f(x,y) = \sin(x^2+y^2)$

$dx = .05$

$dy = -.05$

$f_x = \cos(x^2+y^2)\cdot 2x \qquad f_y = \cos(x^2+y^2)\,2y$

$dw \approx f_x\,dx + f_y\,dy$

$\cos(2)\cdot 2(1)\cdot(0.05) + \cos(2)\cdot 2(1)\cdot(-.05) = \boxed{0}$

2. Use the appropriate chain rule to find $\frac{\partial w}{\partial t}=\frac{\partial w}{\partial x}\frac{\partial x}{\partial t}+\frac{\partial w}{\partial y}\frac{\partial y}{\partial t}+\frac{\partial w}{\partial z}\frac{\partial z}{\partial t}, \frac{\partial w}{\partial s}=\frac{\partial w}{\partial x}\frac{\partial x}{\partial s}+\frac{\partial w}{\partial y}\frac{\partial y}{\partial s}+\frac{\partial w}{\partial z}\frac{\partial z}{\partial s}$ for the function $w=ze^{xy}, x=s-t, y=s+t, z=st$. (8 points)

$\frac{\partial w}{\partial x} = ze^{xy}\cdot y = st(s+t)e^{s^2-t^2} \qquad \frac{\partial x}{\partial t} = -1 \qquad \frac{\partial x}{\partial s} = 1$

$\frac{\partial w}{\partial y} = ze^{xy}\cdot x = st(s-t)e^{s^2-t^2} \qquad \frac{\partial y}{\partial t} = 1 \qquad \frac{\partial y}{\partial s} = 1$

$\frac{\partial w}{\partial z} = e^{xy} = e^{s^2-t^2} \qquad \frac{\partial z}{\partial t} = s \qquad \frac{\partial z}{\partial s} = t$

$\frac{\partial w}{\partial t} = st(s+t)e^{s^2-t^2}\cdot(-1) + st(s-t)e^{s^2-t^2}(1) + e^{s^2-t^2}\cdot s$

$= se^{s^2-t^2}\left[\cancel{-st}-t^2\cancel{+st}-t^2+1\right] = \boxed{se^{s^2-t^2}[1-2t^2]}$

$\frac{\partial w}{\partial s} = st(s+t)e^{s^2-t^2}(1) + st(s-t)e^{s^2-t^2}(1) + e^{s^2-t^2}(t)$

$= te^{s^2-t^2}\left[s^2+\cancel{st}+s^2-\cancel{st}+1\right] = \boxed{te^{s^2-t^2}[1+2s^2]}$