Name KEY

Math 254, Quiz #15, Winter 2012

Instructions: Show all work. Use exact answers unless directed otherwise (with the exception of some application problems). Problems with answers only will rarely receive full credit. Be sure to read each problem carefully and complete all parts.

1. Find the potential function, if it exists for the vector field $\vec{F}(x,y,z)=3x^2y^2\hat{i}+2x^3y\hat{j}+\hat{k}$ or show that the field is not conservative. (5 points)

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 3x^2y^2 & 2x^3y & 1 \end{vmatrix} = (0-0)\hat{i}-(0-0)\hat{j}+(6x^2y-6x^2y)\hat{k}=\vec{0}$$

conservative $\Rightarrow$ $f(x,y,z)=x^3y^2+z+C$

$\int 3x^2y^2\,dx = x^3y^2+g(y,z)$ $\int 2x^3y\,dy = x^3y^2+h(x,z)$ $\int 1\,dz = z+j(x,y)$

2. Evaluate the line integral $\int_C 3(x-y)ds$ along the path $C:\vec{r}(t)=t\hat{i}+(2-t^2)\hat{j}, 0\le t\le 2$. (5 points)

$\vec{r}'(t)=\hat{i}-2t\hat{j}$

$\|\vec{r}'(t)\|=\sqrt{1+4t^2}$

$$3\int_0^2\left[t-(2-t^2)\right]\sqrt{1+4t^2}\,dt = 3\int_0^2 (t^2+t-2)\sqrt{1+4t^2}\,dt$$

$\approx 4.935\ldots \times 3 = 14.806\ldots$

3. Evaluate the line integral $\int_C \vec{F}\cdot d\vec{r}$ for the function $\vec{F}(x,y)=2xy\hat{i}+x^2\hat{j}$ along the path $C:\vec{r}(t)=t^2\hat{i}+(4-2t+t^3)\hat{j}, 0\le t\le 2$. [Hint: If the field is conservative, another path may be easier to evaluate.] (5 points)

~~$4-2(0)+2^3$~~

$2x=2x$ conservative

$f(x,y)=x^2y$

$$\int_C F\cdot dr = x^2y\Big|_{(0,4)}^{(4,8)} = 4^2\cdot 8-0 = 128$$