Name KEY

Math 254, Quiz #13, Winter 2012

Instructions: Show all work. Use exact answers unless directed otherwise (with the exception of some application problems). Problems with answers only will rarely receive full credit. Be sure to read each problem carefully and complete all parts.

1. Find the volume of the solid inside the sphere $x^2+y^2+z^2=16$ and outside the cone (top only) $z=\sqrt{x^2+y^2}$. Use cylindrical coordinates. (7 points)

$= \text{sphere} - \text{inside}$ $\quad \varphi = \pi/4 \quad$ spherical $\quad \rho = 4$

$$\frac{4}{3}\pi(4)^3 - \int_0^{\pi/4}\int_0^{2\pi}\int_0^4 \rho^2 \sin\varphi \, d\rho \, d\theta \, d\varphi =$$

$$\int_0^{\pi/4}\int_0^{2\pi} \frac{1}{3}\rho^3\Big|_0^4 \sin\varphi \, d\theta \, d\varphi \to \int_0^{\pi/4} 2\pi \cdot \frac{1}{3}\cdot 64 \sin\varphi \, d\varphi$$

$$\frac{4}{3}\pi(64) - \left[128\pi \cdot \frac{1}{3}(-\cos\varphi)\right]_0^{\pi/4} = \frac{256}{3}\pi + \frac{128}{3}\pi\left(-\frac{1}{\sqrt{2}}+1\right) = \boxed{\frac{128\pi}{3\sqrt{2}} - \frac{128}{3}\pi}$$

or cylindrical $\quad z=r \quad z=\sqrt{16-r^2} \quad u=16-r^2 \quad \frac{du}{-2}=\cancel{2}r\,dr$

$16-x^2-y^2 = x^2+y^2$

$16 = 2x^2+2y^2$

$8 = x^2+y^2$

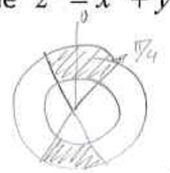
$r = 2\sqrt{2}$

$$\frac{4}{3}\pi(64) \quad \int_0^{2\pi}\int_0^{2\sqrt{2}}\int_r^{\sqrt{16-r^2}} r\,dz\,dr\,d\theta = \frac{256}{3}\pi - \int_0^{2\pi}\int_0^{2\sqrt{2}} r\sqrt{16-r^2} - r^2 \, dr\,d\theta =$$

$$\frac{256}{3}\pi - \int_0^{2\pi}\left[-\frac{1}{3}(16-r^2)^{3/2} - \frac{1}{3}r^3\right]_0^{2\sqrt{2}} d\theta = \frac{256}{3}\pi + 2\pi\left[\frac{1}{3}8^{3/2} + \frac{1}{3}8\cdot\sqrt{8} - \frac{1}{3}16^{3/2}\right] = \boxed{\frac{-128\pi}{3} + \frac{64\sqrt{2}}{3}\pi}$$

$\frac{16\sqrt{2}}{3} \quad \frac{16\sqrt{2}}{3} \quad \frac{1}{3}4^3 = \frac{64}{3} \quad \frac{32\sqrt{2}}{3}$

2. Find the volume of the solid between the spheres $x^2+y^2+z^2=4, x^2+y^2+z^2=9$ and inside the cone $z^2=x^2+y^2$. Use spherical coordinates. (8 points)



$$2\int_0^{\pi/4}\int_0^{2\pi}\int_2^3 \rho^2 \sin\varphi \, d\rho \, d\theta \, d\varphi$$

$$2\int_0^{\pi/4} 2\pi \, \frac{1}{3}\rho^3\Big|_2^3 \sin\varphi \, d\varphi$$

$$\frac{4\pi}{3}\int_0^{\pi/4}(27-8)\sin\varphi \, d\varphi$$

$$\frac{76\pi}{3}(-\cos\varphi)\Big|_0^{\pi/4} = \boxed{\frac{76\pi}{3}\left(1-\frac{1}{\sqrt{2}}\right)}$$