Name $\underline{\quad KEY \quad}$

Math 254, Quiz #10, Winter 2012

Instructions: Show all work. Use exact answers unless directed otherwise (with the exception of some application problems). Problems with answers only will rarely receive full credit. Be sure to read each problem carefully and complete all parts.

1. Integrate. For each integral, sketch the graph of the region. You'll probably want to change to polar coordinates. (4 points each)

   a. $\displaystyle\int_{-2}^{2}\int_{0}^{\sqrt{4-x^2}} (x^2+y^2)\,dy\,dx$

$x^2+y^2=4$

$$\int_0^{\pi}\int_0^{2} r^3\,dr\,d\theta = \int_0^{\pi} \frac{1}{4}r^4\Big|_0^2 d\theta$$

$$= \int_0^{\pi} 4\,d\theta = \boxed{4\pi}$$

   b. Find the area of one petal of the graph $r = 4\cos 2\theta$ using a double integral.

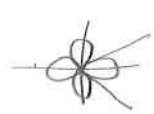
$r = 0 = 4\cos 2\theta$

$\cos 2\theta = 0$

$\frac{2\theta}{2} = \frac{\pi/2}{2}, \frac{3\pi/2}{2}, \frac{-\pi/2}{2}$

$\theta = \pi/4, -\pi/4$

$$\int_{-\pi/4}^{\pi/4}\int_0^{4\cos 2\theta} r\,dr\,d\theta =$$



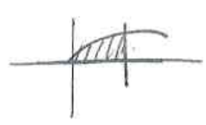
$$\int_{-\pi/4}^{\pi/4} \frac{1}{2}r^2\Big|_0^{4\cos 2\theta} d\theta =$$

$$\int_{-\pi/4}^{\pi/4} \frac{1}{2}(4\cos 2\theta)^2\,d\theta = \int_{-\pi/4}^{\pi/4} 8\cos^2 2\theta\,d\theta$$

$$= 8\cdot\frac{1}{2}\int_{-\pi/4}^{\pi/4} 1+\cos 4\theta\,d\theta = 8\int_0^{\pi/4} 1+\cos 4\theta\,d\theta$$

$$8\theta + \frac{8}{4}\sin 4\theta\Big|_0^{\pi/4} = 2\pi + \cancel{2\sin\pi} = \boxed{2\pi}$$

2. Find the center of mass of the lamina with density $\rho(x,y) = kxy$ bounded by the region $y=\sqrt{x}, y=0, x=1$. (7 points)



$$M = \int_0^1\int_0^{\sqrt{x}} kxy\,dy\,dx = \int_0^1 \frac{k}{2}xy^2\Big|_0^{\sqrt{x}} dx$$

$$= \int_0^1 \frac{k}{2}x^2\,dx = \frac{k}{6}x^3\Big|_0^1 = \frac{k}{6}$$

$$M_y = \int_0^1\int_0^{\sqrt{x}} kx^2y\,dy\,dx = \int_0^1 \frac{k}{2}x^2y^2\Big|_0^{\sqrt{x}} dx = \int_0^1 \frac{k}{2}x^3\,dx = \frac{k}{8}x^4\Big|_0^1 = \frac{k}{8}$$

$$M_x = \int_0^1\int_0^{\sqrt{x}} kxy^2\,dy\,dx = \int_0^1 \frac{k}{3}xy^3\Big|_0^{\sqrt{x}} dx = \int_0^1 \frac{k}{3}x^{5/2}\,dx = \frac{2k}{21}x^{7/2}\Big|_0^1 = \frac{2k}{21}$$

$$\frac{M_x}{M} = \bar{y} = \frac{2k}{21}\cdot\frac{6}{k} = \frac{4}{7} \qquad \frac{M_y}{M} = \frac{k}{8}\cdot\frac{6}{k} = \frac{3}{4} = \bar{x}$$

$$\boxed{(\bar{x},\bar{y}) = \left(\frac{3}{4}, \frac{4}{7}\right)}$$