Name KEY

Math 254, Exam #3, Winter 2012

Instructions: Show all work. Use exact answers unless directed otherwise (with the exception of some application problems). Problems with answers only will rarely receive full credit. Be sure to read each problem carefully and complete all parts.

1. Set up an integral to find the volume of the solid that is the common interior below the sphere $x^2+y^2+z^2=80$ and above the paraboloid $z=\frac{1}{2}(x^2+y^2)$. [Hint: you may want to work in a different coordinate system than rectangular.] (10 points)

$2z+z^2=80$

$1+2z+z^2=81$

$(z+1)^2=9$

$z+1=\pm 3$

$z=2,\cancel{-4}$

$4=x^2+y^2$

$$\int_0^{2\pi}\int_0^2\int_{\frac{1}{2}r^2}^{\sqrt{80-r^2}} r\,dz\,dr\,d\theta$$

$$\int_0^{2\pi}\int_0^2 r\sqrt{80-r^2}-\frac{1}{2}r^3\,dr\,d\theta=$$

$$\int_0^{2\pi}\left[-\frac{1}{2}\frac{2}{3}(80-r^2)^{3/2}-\frac{1}{8}r^4\right|_0^2 d\theta$$

$$\int_0^{2\pi} -\frac{1}{3}(64)^{3/2}-\frac{1}{8}(16)+\frac{1}{3}(80)^{3/2}+0\,d\theta$$

$$=\frac{4(160\sqrt{5}+253)}{3}\pi \approx 814.361\pi$$

2. Integrate. (10 points each)

a. $\int_1^4\int_1^{e^2}\int_0^{1/xz} \ln z\,dy\,dz\,dx$

$= 4\ln 2$

4. Find the center of mass of a solid of density $\rho(x,y,z) = kxyz^2$ over the region bounded by $z = 9 - x^2, y = x, x = 3$ in the first octant. (25 points)

$$M = \int_0^3 \int_0^x \int_0^{9-x^2} kxyz^2 \, dz \, dy \, dx = \frac{19683}{80} k$$

$$M_{xy} = \int_0^3 \int_0^x \int_0^{9-x^2} kxyz^3 \, dz \, dy \, dx = \frac{177147}{160} k$$

$$M_{xz} = \int_0^3 \int_0^x \int_0^{9-x^2} kxy^2z^2 \, dz \, dy \, dx = \frac{104976}{385} k$$

$$M_{yz} = \int_0^3 \int_0^x \int_0^{9-x^2} kx^2yz^2 \, dz \, dy \, dx = \frac{157464}{385}$$

$\bar{x} =$

$$\frac{M_{yz}}{M} = \frac{80}{19683} \cdot \frac{157464}{385} = \boxed{\frac{128}{77}}$$

$\bar{y} =$

$$\frac{M_{xz}}{M} = \frac{80}{19683} \cdot \frac{104976}{385} = \boxed{\frac{256}{231}}$$

$\bar{z} =$

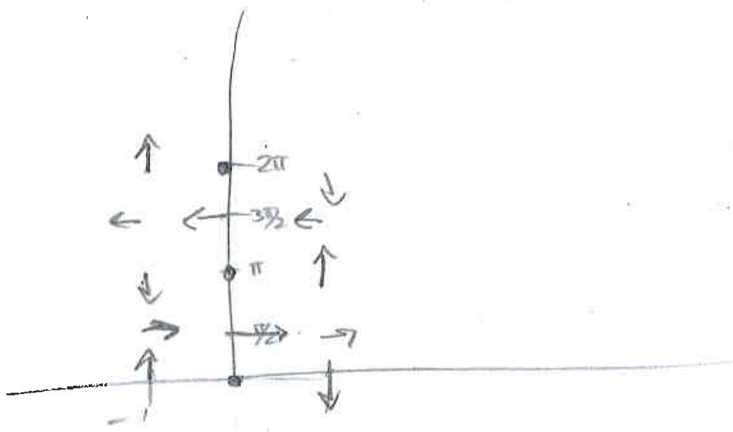
$$\frac{M_{xy}}{M} = \frac{177147}{160} \cdot \frac{80}{19683} = \boxed{\frac{9}{2}}$$

$$\left(\frac{128}{77}, \frac{256}{231}, \frac{9}{2} \right)$$

1.66, 1.108, 4.5

6. Sketch the vector field $F(x,y)=\sin y\vec{i}-x\cos y\vec{j}$ using at least 15 points. (10 points)

x	y	$\vec{F}$
0	0	$\langle 0, 0\rangle$
0	$\pi/2$	$\langle 1, 0\rangle$
0	π	$\langle 0, 0\rangle$
0	$3\pi/2$	$\langle -1, 0\rangle$
0	2π	$\langle 0, 0\rangle$
1	0	$\langle 0, -1\rangle$
1	$\pi/2$	$\langle 1, 0\rangle$
1	π	$\langle 0, 1\rangle$
1	$3\pi/2$	$\langle -1, 0\rangle$
1	2π	$\langle 0, -1\rangle$
-1	0	$\langle 0, 1\rangle$
-1	$\pi/2$	$\langle 1, 0\rangle$
-1	π	$\langle 0, -1\rangle$
-1	$3\pi/2$	$\langle -1, 0\rangle$
-1	2π	$\langle 0, 1\rangle$



7. Determine if the vector field $F(x,y,z)=\sin y\vec{i}-x\cos y\vec{j}+\vec{k}$ is conservative. If it is, find a potential function. (10 points)

$$\frac{\partial M}{\partial y} \qquad \frac{\partial N}{\partial x}$$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ \sin y & -x\cos y & 1 \end{vmatrix} = (0-0)\hat{i}-(0-0)\hat{j}+(-\cos y-\cos y)\hat{k}$$

$$\neq 0$$

not conservative

9. Find the value of the line integral given by

$\int_C \vec{F} \cdot d\vec{r}, F(x,y,z) = yz\vec{i} + xz\vec{j} + xy\vec{k}, C: line\ from\ (0,0,0)\ to\ (5,3,2)$. (10 points)

$f(x,y,z) = xyz$

$$\int_C F \cdot dr = xyz \Big|_{(0,0,0)}^{(5,3,2)} = 5 \cdot 3 \cdot 2 - 0 = \boxed{30}$$

10. Evaluate $\int_C \cos y dx + (xy + x \sin y) dy$ boundary of region between $y = x, y = \sqrt{x}$ (10 points)

$-\frac{\partial M}{\partial y} + \frac{\partial N}{\partial x}$

$y + \sin y + \cancel{\sin y} = y + 2 \sin y$

$$\int_0^1 \int_x^{\sqrt{x}} y + 2\sin y \, dy\, dx$$

$\frac{y^2}{2} \Big|_x^{\sqrt{x}} = \frac{x}{2} - \frac{x^2}{2}$ $\quad 2\int_0^1 -\cos y \Big|_x^{\sqrt{x}} = +2\cos x - 2\cos\sqrt{x}$

$$\int_0^1 \frac{1}{2}x - \frac{1}{2}x^2 \, dx = \frac{1}{4}x^2 - \frac{1}{6}x^3 \Big|_0^1 = \frac{1}{4} - \frac{1}{6} = \frac{3}{12} - \frac{2}{12} = \boxed{\frac{1}{12}}$$

$$\int_0^1 2\cos x - 2\cos\sqrt{x} \, dx = -4\cos(1) - 2\sin(1) + 4$$

$= .239..$