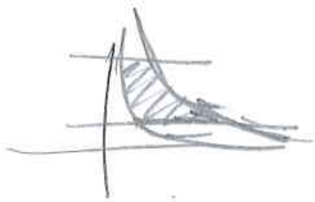
b. $\int_0^{\pi/4}\int_0^{\pi/4}\int_0^{\cos\theta} \rho^2 \sin\varphi \cos\varphi d\rho d\theta d\varphi$

$\frac{5\sqrt{2}}{144}$

3. The value of the integral $I = \int_{-\infty}^{\infty} e^{-x^2/2} dx$ is required in the development of the normal probability density. A) use polar coordinates to evaluate the improper integral $I^2 = \left(\int_{-\infty}^{\infty} e^{-x^2/2} dx\right)\left(\int_{-\infty}^{\infty} e^{-y^2/2} dy\right) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-(x^2+y^2)/2} dA$. B) Use that information to determine *I*. (10 points)

$\sqrt{2\pi}$

5. Find the value of the integral $\iint_R y \sin xy\,dA$ over the region bounded by $xy = 1, xy = 4, y = 4, y = 1$. You may use the change of variables $x = \frac{u}{v}, y = v$. Sketch the region before and after the change of variables, find the Jacobian, and evaluate the integral. (20 points)



$$\frac{\partial(x,y)}{\partial(u,v)} = \begin{vmatrix} \partial x/\partial u & \partial x/\partial v \\ \partial y/\partial u & \partial y/\partial v \end{vmatrix} = \begin{vmatrix} 1/v & -\frac{u}{v^2} \\ 0 & 1 \end{vmatrix} = \frac{1}{v} - 0 = \frac{1}{v}$$

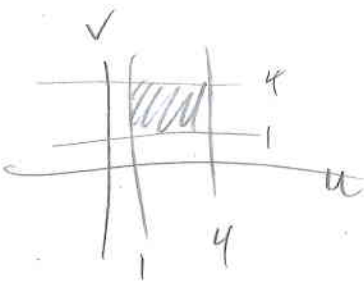
$xy = u$

$v = y$

$$\int_1^4 \int_1^4 \cancel{v} \sin u \cdot \frac{1}{\cancel{v}}\, du\, dv = -\int_1^4 \cos 4 - \cos 1 \, dv =$$

$$\boxed{-3(\cos 4 - \cos 1)}$$

✓



8. For each vector field $\vec{F}$ below, find $\vec{\nabla}\times\vec{F}$ and $\vec{\nabla}\cdot\vec{F}$. (10 points each)

a. $\vec{F}(x,y,z) = xyz\hat{i} + x^2y\hat{j} + yz^2\hat{k}$

$$\vec{\nabla}\times\vec{F} \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ xyz & x^2y & yz^2 \end{vmatrix} = (z^2-0)\hat{i} - (0-xy)\hat{j} + (2xy - yz)\hat{k}$$

$$= z^2\hat{i} + xy\hat{j} + (2xy - yz)\hat{k}$$

$$\vec{\nabla}\cdot F = yz + x^2 + 2yz = 3yz + x^2$$

b. $\vec{F}(r,\theta,z) = \underset{M}{\ln r}\,\widehat{e_r} + \underset{N}{r\cos z}\,\widehat{e_\theta} + \underset{P}{z\tan\theta}\,\widehat{e_z}$. You may use the formulas

$$\nabla\times F = \left(\frac{1}{r}\cdot\frac{\partial P}{\partial\theta} - \frac{\partial N}{\partial z}\right)\widehat{e_r} + \left(\frac{\partial M}{\partial z} - \frac{\partial P}{\partial r}\right)\widehat{e_\theta} + \frac{1}{r}\cdot\left(\frac{\partial}{\partial r}\underbrace{[rN]}_{r^2\cos z} - \frac{\partial M}{\partial\theta}\right)\widehat{e_z}$$ and

$$\vec{\nabla}\cdot\vec{F} = \frac{1}{r}\cdot\frac{\partial}{\partial r}\underbrace{[rM]}_{r\ln r} + \frac{1}{r}\cdot\frac{\partial N}{\partial\theta} + \frac{\partial P}{\partial z}.$$

$$\vec{\nabla}\times\vec{F} \left\langle \frac{1}{r}z\sec^2\theta + r\sin z,\ 0-0,\ \frac{1}{r}(2r\cos z - 0) \right\rangle =$$

$$\boxed{\left(\frac{z}{r}\sec^2\theta + r\sin z\right)\hat{e}_r \cdot (2\cos z)\hat{e}_z \rangle}$$

$$\vec{\nabla}\cdot\vec{F} = \frac{1}{r}\left(\ln r + r\cdot\frac{1}{r}\right) + \frac{1}{r}(0) + \tan\theta$$

$$= \boxed{\frac{\ln r}{r} + \frac{1}{r} + \tan\theta}$$

11. Find the value of the line integral given by

$$\int_C \vec{F}\cdot d\vec{r}, F(x,y,z) = ye^{xy}\vec{i} + xe^{xy}\vec{j}, C: line\ from\ (0,0)\ to\ (0,3)\ to\ (3,3)\ to\ (3,0)\ to\ (0,0).$$

[Hint: if the field is conservative, there may be an easier way. If you go this route, show work that proves conservativeness and state the theorem that allows you to give your answer. Otherwise, parameterize or apply an appropriate theorem.] (15 points)

$$\int ye^{xy}\,dx = e^{xy} + g(y)$$

$$\int xe^{xy}\,dy = e^{xy} + h(x)$$

$$f(x,y) = e^{xy}$$

Closed loop in conservative field $= 0$

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Can also do by Green's theorem

$$\frac{\partial[xe^{xy}]}{\partial y} = xye^{xy}$$

$$\frac{\partial[ye^{xy}]}{\partial x} = \frac{xye^{xy}}{0}$$

$$\int_0^3\int_0^3 0\,dA = 0$$