Name KEY

Math 254, Exam #2, Winter 2012

Instructions: Show all work. Use exact answers unless directed otherwise (with the exception of some application problems). Problems with answers only will rarely receive full credit. Be sure to read each problem carefully and complete all parts.

1. Find the equation to the tangent plane to the graph $x^2+y^2+z=9$ at the point $P(1,2,4)$. (7 points)

$F(x,y,z) = x^2+y^2+z-9$

$\nabla F = \langle 2x, 2y, 1\rangle$

$\nabla F(1,2,4) = \langle 2,4,1\rangle$

plane: $\boxed{2(x-1)+4(y-2)+1(z-4)=0}$

2. What is the normal line to the graph above at the same point in parametric form. (5 points)

$$\boxed{\vec{r}(t) = (2t+1)\hat{i} + (4t+2)\hat{j} + (t+4)\hat{k}}$$

btw: Symmetric form is $\frac{x-1}{2} = \frac{y-2}{4} = \frac{z-4}{1}$

5. Find the absolute extrema for the function $f(x,y) = 12 - 3x - 2y$ on the region bounded by the triangle with vertices (2,0), (0,1) and (1,2). Sketch the graph of the region. (10 points)

$m = \frac{2-0}{1-2} = \frac{2}{-1} = -2$

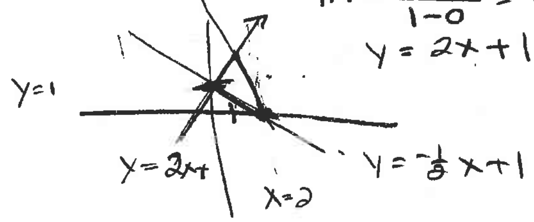
$y = -2(x-2)$
$= -2x + 4$

$m = \frac{1-0}{0-2} = \frac{1}{-2} = -\frac{1}{2}$

$\boxed{y = -\frac{1}{2}x + 1}$

$m = \frac{2-1}{1-0} = \frac{2}{1}$

$y = 2x + 1$



$f_x = -3 \neq 0$

$f_y = -2 \neq 0$ no critical point

corner points

(2,0), (0,1), (1,2)

$f(x, -2x+4) = 12 - 3x - 2(-2x+4) =$
$12 - 3x + 4x - 8 = 4 + x$

$f'(x, -2x+4) = 1 \neq 0$

$f(x, 2x+1) = 12 - 3x - 2(2x+1) =$
$12 - 3x - 4x - 2 = 10 - 7x$

$f'(x, 2x+1) = -7 \neq 0$

$f(x, -\frac{1}{2}x+1) = 12 - 3x - 2(-\frac{1}{2}x+1) =$
$12 - 3x + x - 2$
$= 10 - 2x$

$f'(x, -\frac{1}{2}x+1) = -2 \neq 0$

no critical points on boundaries

$f(2,0) = 12 - 3(2) - 2(0) =$
$12 - 6 = 6$

$f(0,1) = 12 - 3(0) - 2(1) =$
$12 - 2 = \boxed{10}$ MAX

$f(1,2) = 12 - 3(1) - 2(2)$
$12 - 3 - 4 = \boxed{5}$ MIN

6. Let $T(x,y,z) = 100 + x^2 + y^2$ represent the temperature at each point on the sphere $x^2 + y^2 + z^2 = 50$. Find the maximum temperature on the curve formed by the intersection of the sphere and the plane $x - z = 0$. (10 points)

$x = z$
$2x^2 + y^2 = 50$
$2x^2 + y^2 - 50 = 0$

$F(x,y,z,\lambda) = 100 + x^2 + y^2 - \lambda(2x^2 + y^2 - 50)$

$F_x = 2x - 2\lambda x = 0 \qquad 2x(1-\lambda) = 0$

$F_y = 2y - 2\lambda y = 0 \qquad 2y(1-\lambda) = 0$

$F_\lambda = 2x^2 + y^2 = 50 \qquad x = 0, y = 0$ or $\cancel{\lambda = 1}$

$x = 0 \Rightarrow y^2 = 50 \Rightarrow y = \pm 5\sqrt{2} \qquad (0, \pm 5\sqrt{2})$

$y = 0 \Rightarrow 2x^2 = 50 \Rightarrow x^2 = 25 \qquad x = \pm 5 \qquad (\pm 5, 0)$

$T(0, \pm 5\sqrt{2}, z) = 100 + 0^2 + 50 = \boxed{150}$ ← MAX

$T(\pm 5, 0, z) = 100 + 25 + 0 = 125$

9. Integrate.

a. $\int_{e^y}^{y} \frac{y \ln x}{x} dx$ (7 points)

$u = \ln x$

$du = \frac{1}{x} dx$

$y \int u \, du = \frac{u^2}{2} = y\frac{(\ln x)^2}{2}\Big|_{e^y}^{y} =$

$\frac{y}{2}\left[(\ln y)^2 - (\ln e^y)^2\right] = \boxed{\frac{y}{2}\left[(\ln y)^2 - y^2\right]}$

b. $\int_0^2 \int_{3y^2-6y}^{2y-y^2} 3y\,dx\,dy$ (10 points) $\int_0^2 3xy \Big|_{3y^2-6y}^{2y-y^2} dy =$

$\int_0^2 3(2y-y^2)y - 3(3y^2-6y)y \, dy = \int_0^2 6y^2 - 3y^3 - 9y^3 + 18y^2 \, dy =$

$\int_0^2 24y^2 - 12y^3 \, dy = 8y^3 - 3y^4 \Big|_0^2 = 8(2)^3 - 3(2)^4 =$

$64 - 48 = \boxed{16}$

c. $\int_0^2 \int_y^{\sqrt{8-y^2}} \sqrt{x^2+y^2}\,dx\,dy$ (10 points)

Convert to polar

$\int_0^{\pi/4} \int_0^{2\sqrt{2}} r^2 \, dr \, d\theta$

$= \int_0^{\pi/4} \frac{r^3}{3}\Big|_0^{2\sqrt{2}} d\theta = \int_0^{\pi/4} \frac{16\sqrt{2}}{3} d\theta = \frac{16\sqrt{2}}{3} \cdot \frac{\pi}{4} = \boxed{\frac{4\sqrt{2}\pi}{3}}$