Section 3.2 Graphing Equations in Two Variables

Math 102 Course Outline Unit IV Objectives:

- Graph equations using a graphing utility.
- Approximate x- and y-intercepts of graphs using a graphing utility.

  NOTE: Only the objective "approximate *y*-intercept of a graph" is covered in Section 3.2.

New calculator features used in this lesson:

- Viewing rectangle
- WINDOW key
- Xmin
- Ymin
- Xmax
- Ymax
- Xscl
- Yxcl
- Zoom menu: ZStandard, ZInteger, and Zdecimal
- TRACE key
- VALUE feature

Basics

There are five keys just below the graphing screen: Y=, WINDOW, ZOOM, TRACE, and GRAPH.

- Up to 10 functions can be inputted to the Y= menu. Each is identified by a number: Y1, Y2, Y3, …, Y9, Y10.
- WINDOW sets the size of the viewing screen.
- ZOOM is a menu that allows you to zoom on the graph.
- TRACE allows you to "walk" along the graph.
- GRAPH is used to see the graph f a function.

MODE

- Press the MODE key and use the arrow key to position the cursor over FUNC on the 4th line down. This setting will graph functions on the graph screen. Make sure FUNC is highlighted.

Viewing Rectangle

The viewing rectangle is where an equation is graphed. The calculator uses the value of Xmin that is given in the WINDOW menu to compute the associated y-value of the equation. That Xmin is then incremented by $\frac{\text{Xmax - Xmin}}{94}$ to compute the next *y*-value, and so on, all the way up to Xmax. There are several built-in viewing rectangles.

- Zstandard has Xmin = -10, Xmax = 10, Ymin = -10 and Ymax = 10. We sometimes use the notation [-10, 10, 1] by [-10, 10, 1] to note these dimensions. This notation means that the lowest *x*-value shown in the window is -10, called Xmin, that the highest *x*-value shown in the window is 10, called Xmax, and that each tick mark on the *x* axis is 1 unit, called Xscl.

- ZInteger has Xmin = -47, Xmax = 47, Ymin = -31 and Ymax = 31. The notation [-47,47,1] by [-31, 31, 1] describes these dimensions. This notation means that the lowest *x*-value shown in the window is -47, Xmin, that the highest *x*-value shown in the window is 47, Xmax, and that each tick mark on the *x* axis is 1 unit.
    - ❖ Note: Because Xmin – Xmax = 94, a function graphed in this viewing rectangle is incremented in 1-unit steps.

- Zdecimal has Xmin = -4.7, Xmax = 4.7, Ymin = -3.1 and Ymax = 3.1, or [-4.7, 4.7, 1] by [-3.1, 3.1, 1]. The lowest *x*-value shown in the window is -4.7, Xmin, that the highest *x*-value shown in the window is 4.7, Xmax, and that each tick mark on the *x* axis is 1 unit.
    - ❖ Note: Because Xmin – Xmax = 9.4, the graph is incremented in 0.1-unit steps.

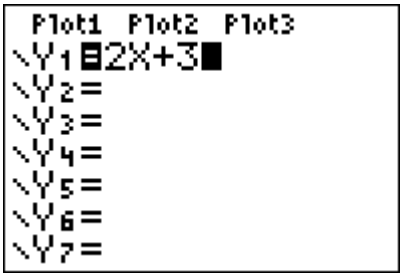
To access the standard window where (0,0) is at the center, press ZOOM 6.
To access the integer window where (0, 0) is at the center, press ZOOM 6 ZOOM 8 ENTER.
To access the decimal window where (0, 0) is at the center, press ZOOM 4.

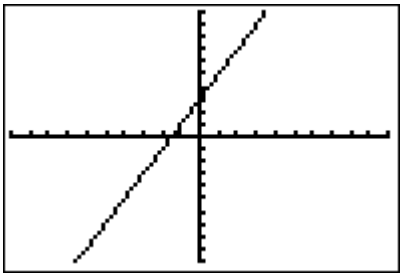
To determine a graphing calculator window that accurately displays the *complete graph* of an equation in two variables, students must be aware of the solutions of the equation in two variables, especially the intercepts. Once again, a linear equation must be in slope-intercept form, and non-linear equations must be solved for *y*.

A function must be *on* to be graphed. A function is *on* when the equal sign is highlighted. To toggle between *on* and *off*, use the arrow keys to position the cursor over the equal sign. Press ENTER. The function will now be *on* if was *off*, and *off* it was *on*.

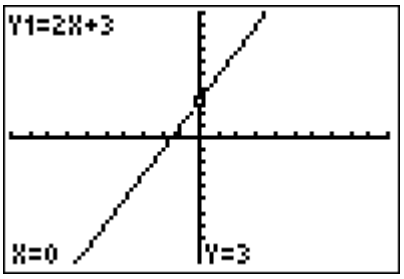
Example 1: Graph the equation $y = 2x + 3$ on the calculator in the standard window.

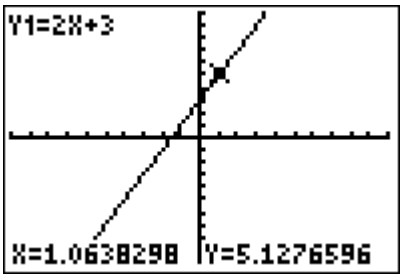
Solution: Enter the equation $y = 2x + 3$ into the Y= screen. Press ZOOM 6. The following screens should be seen.



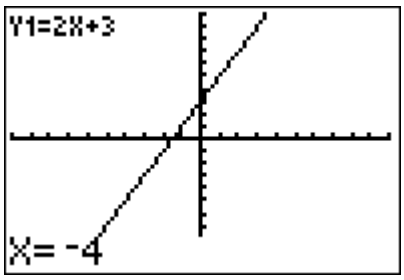


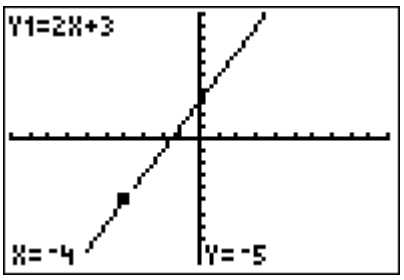
Use the TRACE key to identify some ordered pair solutions of the equation $y = 2x + 3$. The TRACE key is located below the graphing screen. When you initially press the TRACE key, you see the *y*-intercept, (0, 3) identified on the graph. Now use the left and right arrow keys to "walk" along the graph.



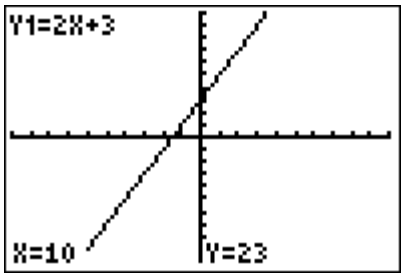


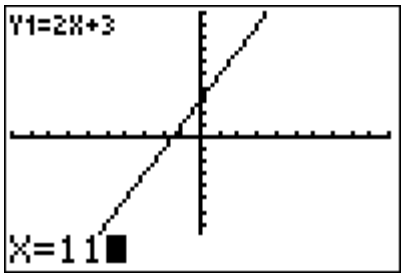
In the Standard window, $\frac{\text{Xmax - Xmin}}{94}$ frequently provides a decimal number. So to find ordered pairs for which the *x*-value is "nice," use the VALUE feature of the calculator. For example, if you wish to find the value of *y* when *x* = -4, while in TRACE, just type in the number -4 and press ENTER.

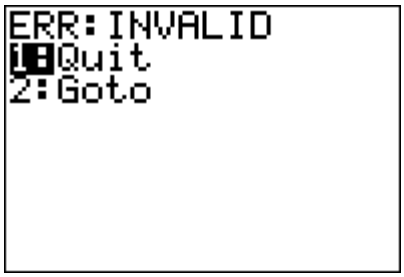




The point whose coordinates are (-4, -5) are identified on the graph. The VALUE feature will work as long as the value of *x* lies between the Xmin and Xmax values of the given viewing rectangle. So in the Standard window you can find the value of *y* when *x* = 10, but you can't find the value of *y* when *x* = 11. Notice that the value of *y* when *x* = 10 is off the graphing screen, but it's still identified.

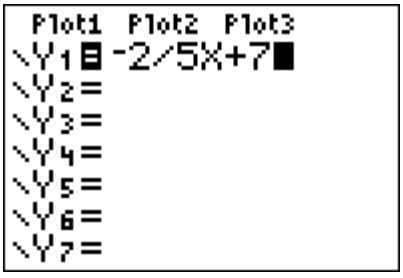


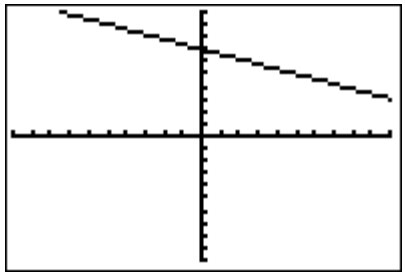




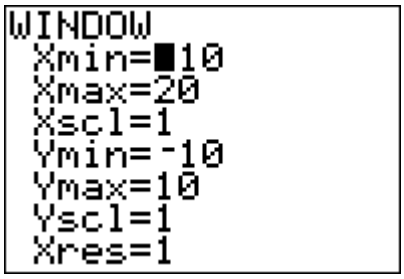
Example 2: Graph the equation $y = -\frac{2}{5}x + 7$ on the calculator in the standard window.

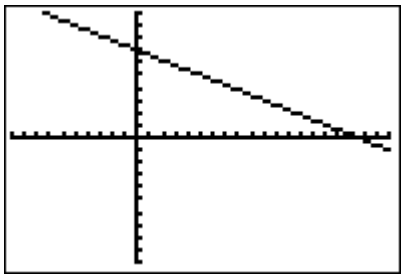
Solution: Enter the equation $y = -\frac{2}{5}x + 7$ into the Y= screen. Press ZOOM 6. The following graph should be seen.





We like to see a "complete" graph – one in which both intercepts are clearly seen in the graphing screen. Because the *x*-intercept appears to be off the screen to the right, we need to change the Xmax to a number greater than 10. Let's try 20. After changing the window so that Xmax = 20, press the GRAPH key. The graph $y = -\frac{2}{5}x + 7$ of appears to the right, below.





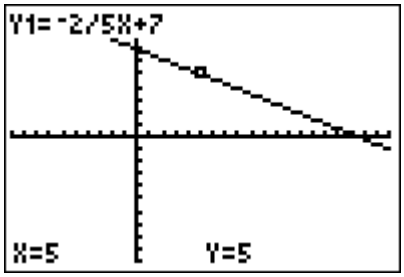
INSTRUCTOR NOTE:

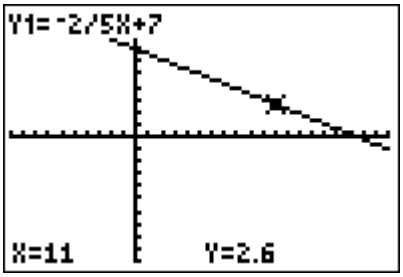
A quick way to find a suitable viewing rectangle is to know both intercepts of the graph. Because the equation $y = -\frac{2}{5}x + 7$ is in slope-intercept form, emphasize that students know that the *y*-intercept is 7. (Also, when a function is graphed in the Standard viewing rectangle centered at (0, 0), when the TRACE key is pressed, the y-intercept will always be the first point identified on the graph.) To find the *x*-intercept of $y = -\frac{2}{5}x + 7$, let $y = 0$ and solve for *x*.

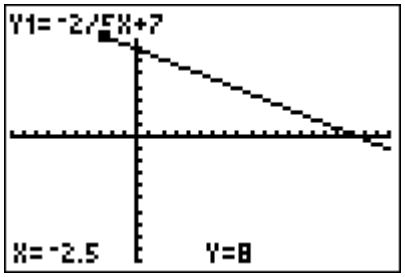
$$y = -\frac{2}{5}x + 7$$
$$0 = -\frac{2}{5}x + 7$$
$$5(0) = 5\left(-\frac{2}{5}x + 7\right)$$
$$0 = -2x + 35$$
$$2x = 35$$
$$x = 17.5$$

After finding the *x*-intercept algebraically, we know that two points on the graph are (0, 7) and (17.5, 0). Knowing these two coordinates assists us in finding a viewing rectangle that shows the complete graph of $y = -\frac{2}{5}x + 7$.

Press the TRACE key to identify ordered pair solutions of the equation $y = -\frac{2}{5}x + 7$. In this case, because our graph isn't centered at (0, 0), the first point identified on the graph is (5, 5). Use the VALUE feature to find other ordered pair solutions of $y = -\frac{2}{5}x + 7$.







Example 3: The Crickets are the newest band in town and have a gig performing at a local concert. They agree that their total fee will be $500 plus an additional $3.00 per person who attends the concert. The equation that calculators the total fee for performing, *F*, is $F = 500 + 3p$, where *p* is the number of people in attendance.

a. Interpret the meaning of each ordered pair (*p*, *F*).

b. Complete the ordered pairs: (0, _____), (50, _____),

(100, _____0, (250, _____).

c. Graph the equation $F = 500 + 3p$ in a suitable viewing rectangle.

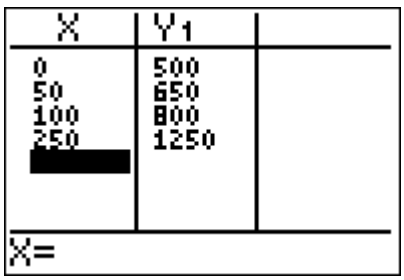
INSTRUCTOR NOTES:

❖ This is a good place to begin talking about the appropriate values of the independent variable, *p*. That is, can you sell a negative number of tickets? Would the band take a negative fee? Lay the groundwork here to talk about domain and range later.

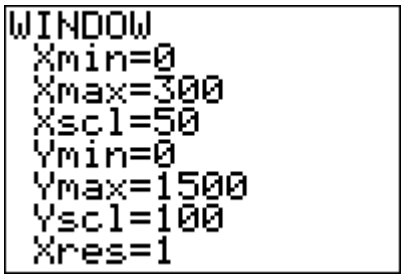
❖ Also talk about setting the Xscl and Yscl values. For example, if Ymax is set at 750 and Yscl is 1, then the *y*-axis will appear as a very dark line, or even as two lines. So setting Yscl at 50 or 100 will produce a clearer graph.

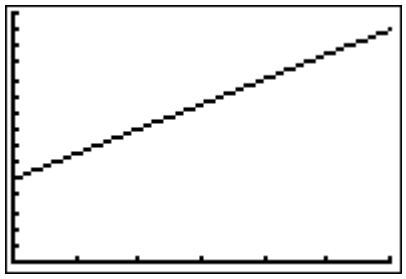
Solution: a. (number of people in attendance, fee)

b.



c.

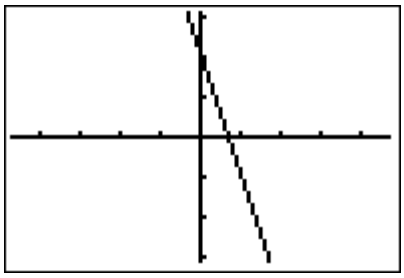


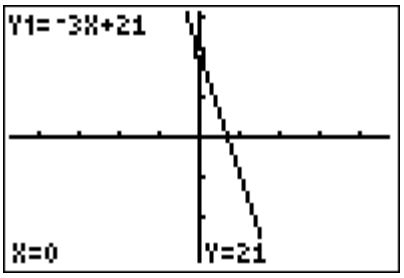


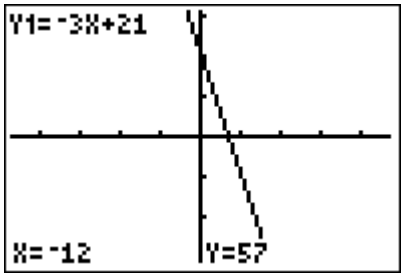
Other commonly-used viewing rectangles are ZInteger (Zoom 8) and ZDecimal (Zoom 4).

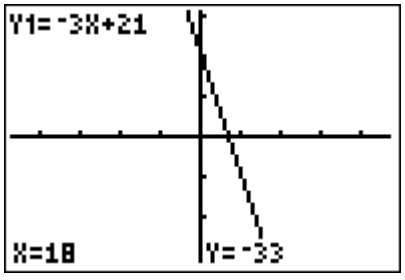
Example 4: Graph the equation $y = -3x + 21$ in the Integer window. Complete the ordered pairs: (0, _____), (-12, _____), (18, _____), (21.75, _____)

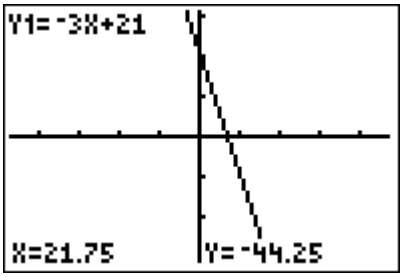
Solution: For the graph to be centered at the origin, press ZOOM 6 ZOOM 8 ENTER, and then put the expression $-3x + 21$ in the Y= editor. The "value" feature of TRACE can be used to complete the ordered pairs.





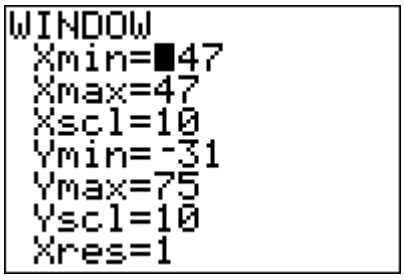


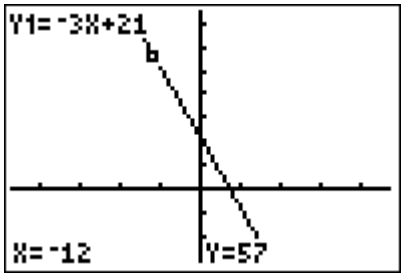




INSTRUCTOR NOTE

If you wish to see a portion of the graph for which Ymax is larger than 31 or Ymin is less than -31, you can change the values of Ymin and Ymax without affecting the "nice" x-values that are produced in the viewing rectangle of the Integer window. For example, for x = -12, y = 57, but y = 57 is off the screen. Change the value of Ymax to 75, and the ordered pair (-12, 57) is clearly seen.





Example 5: Graph the equation $y = -0.5x + 2$ in the Decimal window. Complete the ordered pairs (0, _____), (-1.2, _____), (3.6, _____).

Solution: Remember that this window is [-4.7, 4.7, 1] by [-3.1, 3.1, 1].