MTH 265, Quiz #6, Summer 2024 Name KEY

Instructions: Show all work. Use exact answers unless otherwise asked to round.

1. Find ∇f and $\nabla^2 f$ for the function $f(x,y,z) = \frac{1}{2}xy^2\cos(y+z^3)$.

$$\nabla f = \left\langle \tfrac{1}{2}y^2\cos(y+z^3),\ xy\cos(y+z^3) - \tfrac{1}{2}xy^2\sin(y+z^3),\ -\tfrac{1}{2}xy^2\sin(y+z^3)\,3z^2 \right\rangle$$

$$\nabla^2 f = 0 + x\cos(y+z^3) - xy\sin(y+z^3) - xy\sin(y+z^3) - \tfrac{1}{2}xy^2\cos(y+z^3) - 3xy^2z\sin(y+z^3) - \tfrac{3}{2}xy^2z^2\cos(y+z^3)\,3z^2$$

2. Find $\nabla\cdot\vec{F}$ and $\nabla\times\vec{F}$ for $\vec{F}(x,y,z) = \sin(xy)\,\hat{\imath} - \cos(yz)\,\hat{\jmath} + \tan(xz)\hat{k}$.

$$\vec{\nabla}\cdot F = y\cos xy + z\sin yz + x\sec^2(xz)$$

$$\vec{\nabla}\times\vec{F} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ \partial/\partial y & \partial/\partial y & \partial/\partial z \\ \sin xy & -\cos yz & \tan xz \end{vmatrix} = (0 - y\sin(yz))\hat{\imath} - (z\sec^2 xz - 0)\hat{\jmath} + (0 - x\cos xy)\hat{k}$$

$$= y\sin yz\,\hat{\imath} - z\sec^2 xz\,\hat{\jmath} - x\cos xy\,\hat{k}$$

3. Determine if the vector field $\vec{F}(x,y,z) = (x+y)\hat{\imath} + (y-z)\hat{\jmath} + z^2\hat{k}$ is conservative.

$$\vec{\nabla}\times\vec{F} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ x+y & y-z & z^2 \end{vmatrix} = (0+1)\hat{\imath} - (0-0)\hat{\jmath} + (0-1)\hat{k}$$

$$\langle 1, 0, -1 \rangle$$

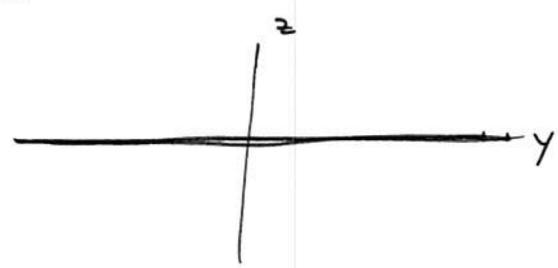
not conservative

4. Consider the function $f(x,y) = \frac{x}{x^2+y^2}$. Sketch the following:

a. The trace on the yz-plane.

$X = 0$

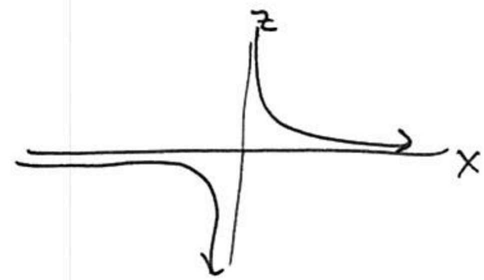
$$z = \frac{0}{0+y^2} = 0$$



b. The trace on the xz-plane.

$y=0$

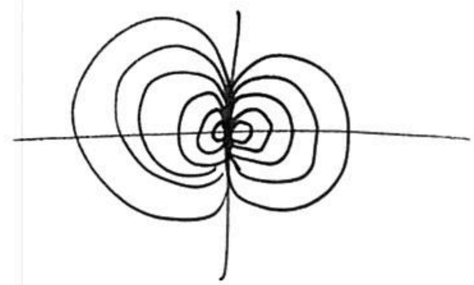
$z = \frac{x}{x^2+0^2} = \frac{x}{x^2} = \frac{1}{x}$



c. 10 level curves.

$x^2+y^2 = \frac{x}{z}$

Circle centered at $\frac{1}{2z} = x$



d. Use technology to verify your level curves and produce a 3D graph of the function to verify your results. Attach the graphs to your submission.

See attached

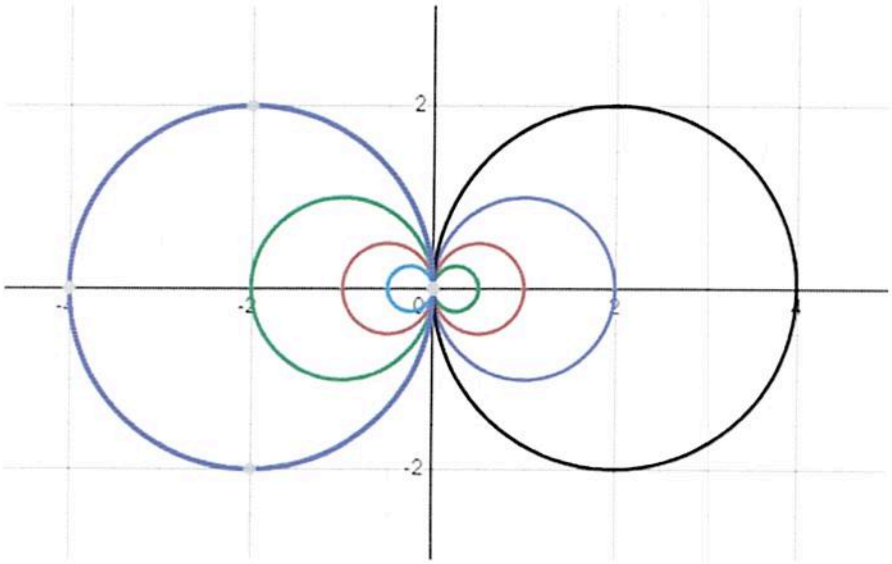
5. Find the potential function, if it exists, for the vector field $\vec{F}(x,y,z) = (2xy+yz^2)\hat{\imath} + (x^2-2yz+xz^2)\hat{\jmath} + (2xyz-y^2+\cos z)\hat{k}$. If not potential function exists, show work to prove that it is not.

$\int 2xy + yz^2\,dx = xy + xyz^2 + f(y,z)$

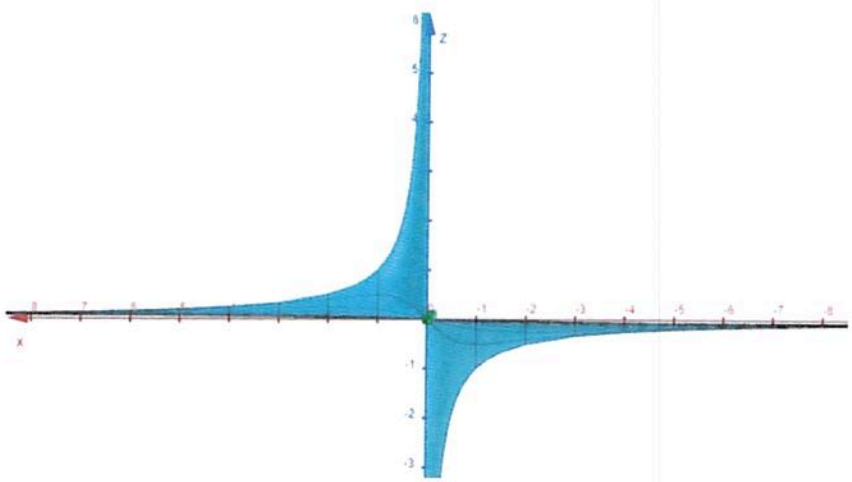
$\int x^2 - 2yz + xz^2\,dy = x^2y - y^2z + xyz^2 + g(x,z)$

$\int 2xyz - y^2 + \cos z\,dz = xyz^2 - zy^2 + \sin z + h(x,y)$

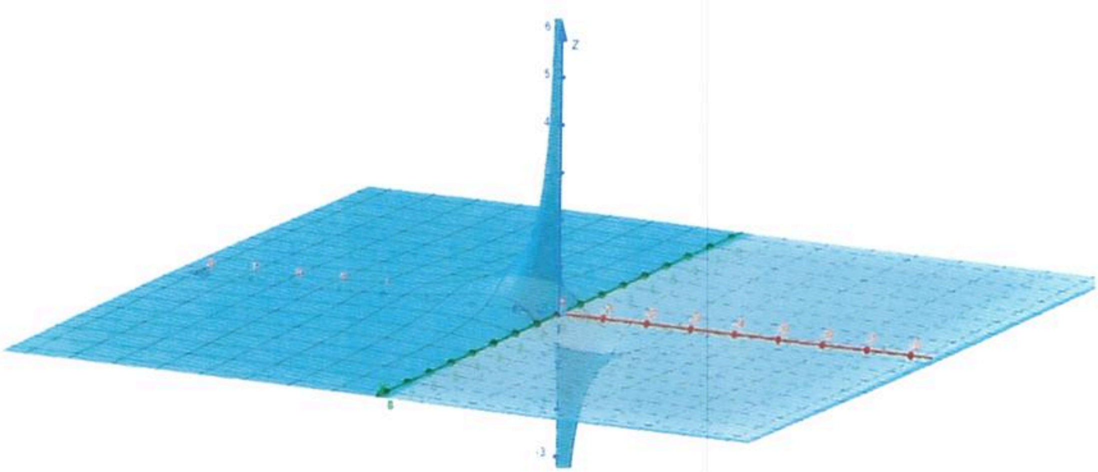
$\varphi(x,y,z) = x^2y + y^2z + xyz^2 + \sin z + K$



2
-2
-4
-2
0
2
4



z
x



z