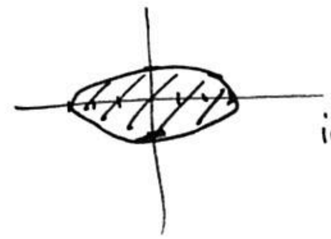
MTH 265, Quiz #3, Summer 2024 Name KEY

Instructions: Show all work. Use exact answers unless otherwise asked to round.

1. Find the domain and range of $f(x, y) = \sqrt{9 - x^2 - 9y^2}$ and sketch the domain in the plane.

$$9 - x^2 - 9y^2 \geq 0$$
$$x^2 + 9y^2 \leq 9$$
$$\frac{x^2}{9} + y^2 \leq 1$$



inside The ellipse

$$D: \{(x,y) \mid x^2 + 9y^2 \leq 9\}$$

max $= 3$
min $= 0$

$$R: [0, 3]$$

2. Consider the function $f(x, y) = \sqrt{x^2 - y^2}$. Convert the function to the indicated coordinate system or format.

$z = \rho \cos\varphi$
$x = \rho \sin\varphi \cos\theta$
$y = \rho \sin\varphi \sin\theta$

a. Write f in spherical coordinates.

$$\sqrt{\rho^2 \sin^2\varphi \cos^2\theta - \rho^2 \sin^2\varphi \sin^2\theta}$$
$$= \cancel{\rho} \sin\varphi \sqrt{\cos 2\theta} = \cancel{\rho}\cos\varphi$$
$$\tan^2\varphi = \cos 2\theta$$

b. Write f in cylindrical coordinates.

$x = r\cos\theta$, $y = r\sin\theta$

$$z = \sqrt{r^2\cos^2\theta - r^2\sin^2\theta}$$
$$z = r\sqrt{\cos 2\theta}$$

c. Write f in parametric surface form, $\vec{r}(u, v)$.

$$r(u, v) = (u\cos v)\hat{\imath} + (u \sin v)\hat{\jmath} + (u\sqrt{\cos 2v})\hat{k}$$

3. Find $\lim\limits_{(x,y)\to(0,0)} \frac{xy^4}{x^2+y^8}$ if it exists or prove that it does not.

let $x^2 = y^8$
$x = ky^4$

$$\lim_{y\to 0} \frac{ky^4 \cdot y^4}{k^2y^8 + y^8} = \lim_{y\to 0} \frac{k\cancel{y^8}}{\cancel{y^8}(k^2+1)} =$$

$$\lim_{y\to 0} \frac{k}{k^2+1} = \text{DNE.}$$