Instructions: Show all work. Use exact answers unless otherwise asked to round.

1. Evaluate the surface integral $\iint_S y^2 dS$ where S is the part of the sphere $x^2 + y^2 + z^2 = 4$ that lies inside the cylinder $x^2 + y^2 = 1$.

$G = z - \sqrt{4-x^2-y^2}$

$z = \sqrt{4-x^2-y^2}$

$\nabla G = \left\langle \dfrac{-x}{\sqrt{4-x^2-y^2}}, \dfrac{-y}{\sqrt{4-x^2-y^2}}, 1 \right\rangle$

$\|\nabla G\| = \sqrt{\dfrac{x^2}{4-x^2-y^2} + \dfrac{y^2}{4-x^2-y^2} + \dfrac{1(4-x^2-y^2)}{4-x^2-y^2}}$

$= \sqrt{\dfrac{4}{4-x^2-y^2}} = \dfrac{2}{\sqrt{4-x^2-y^2}} = \dfrac{2}{\sqrt{4-r^2}}$

$\int_0^{2\pi} \int_0^1 r^2 \sin^2\theta \cdot \dfrac{2}{\sqrt{4-r^2}} \, r \, dr \, d\theta$

$\int_0^{2\pi} \int_0^1 \dfrac{2r^3 \sin^2\theta}{\sqrt{4-r^2}} \, dr \, d\theta = \int_0^{2\pi} \sin^2\theta \left(-\dfrac{2}{3}\right)(x^2+8)\sqrt{4-x^2} \Big|_0^1 \, d\theta =$

$\int_0^{2\pi} -\dfrac{2}{3}(9\sqrt{3}-16)\sin^2\theta \, d\theta = -\dfrac{2}{3}(9\sqrt{3}-16)\int_0^{2\pi} \dfrac{1}{2}(1-\cos 2\theta)\,d\theta =$

$-\dfrac{1}{3}(9\sqrt{3}-16)\int_0^{2\pi} 1 - \cos 2\theta \, d\theta = -\dfrac{1}{3}(9\sqrt{3}-16)\left(\theta - \dfrac{1}{2}\sin 2\theta\right)\Big|_0^{2\pi} =$

$\dfrac{2(9\sqrt{3}-16)\pi}{3}$

2. Find the flux $\iint_S \vec{F} \cdot d\vec{S}$ for $\vec{F}(x,y,z) = xy\hat{\imath} + yz\hat{\jmath} + xz\hat{k}$ where S is the surface of the paraboloid $z = 4 - x^2 - y^2$ that lies above the square $0 \le x \le 1, 0 \le y \le 1$ with upward orientation.

$G = 4 - x^2 - y^2 - z^2$ downward

$\nabla G = \langle -2x, -2y, -1\rangle$

$\vec{F} \cdot \nabla G = 2x^2 y - 2y^2 z - xz = -2x^2 y - 2y^2(4-x^2-y^2) - x(4-x^2-y^2)$

$= 2x^2 y - 8y^2 + 2x^2 y^2 + 2y^4 - 4x + x^3 + xy^2$

upward $= 2x^2 y + 8y^2 - 2x^2 y^2 - 2y^4 + 4x - x^3 - xy^2$

$\int_0^1 \int_0^1 2x^2 y + 8y^2 - 2x^2 y^2 - 2y^4 + 4x - x^3 - xy^2 \, dy \, dx =$

$\int_0^1 = -\dfrac{1}{4}x^4 - \dfrac{2}{3}x^3 (y)(y-1) + 2x^2 + \dfrac{x^2}{2}(y^2-4) + 8xy^2 \Big|_0^1 \, dy$

$\int_0^1 \dfrac{41}{6}y^2 + \dfrac{2}{3}y + \dfrac{7}{4} \, dy = \dfrac{41}{18}y^3 + \dfrac{y^2}{2} + \dfrac{7}{4}y \Big|_0^1 = \dfrac{157}{36}$