

Instructions: Work the problems below as directed. Show all work. Clearly mark your final answers. Use exact values unless the problem specifically directs you to round. Simplify as much as possible. Partial credit is possible, but solutions without work will not receive full credit.

1. Find the antiderivatives of each of the following.

a. $\int e^x - \sin x + 4x^4 dx$

$$e^x + \cos x + \frac{4}{5}x^5 + C$$

b. $\int \frac{\sqrt{x+x}}{x^2} dx$ $\frac{x^{1/2}}{x^2} + \frac{x}{x^2} = x^{-3/2} + \frac{1}{x}$

$$(-2)x^{-1/2} + \ln x + C = -\frac{2}{\sqrt{x}} + \ln x + C$$

c. $\int \frac{1}{1+x^2} - \frac{1}{\sqrt{1-x^2}} + \cosh x dx$

$$\arctan x - \arcsin x + \sinh x + C$$

2. Use summation formulas to find the value of the sums.

a. $\sum_{i=1}^{10} (i^2 - 2i)$

$$\sum i^2 = \frac{n(2n+1)(n+1)}{6}$$

$$\sum i = \frac{n(n+1)}{2}$$

$$385 - 2(55) = 275$$

$$\frac{10(11)(21)}{6} = 385$$

$$10(11)/2 = 55$$

b. $\sum_{j=11}^{20} (j^3 - 4j)$

$$\sum j^3 = \frac{n^2(n+1)^2}{4}$$

$$\sum j = \frac{n(n+1)}{2}$$

$$\sum_{j=11}^{20} f(j) = \sum_{j=1}^{20} f(j) - \sum_{j=1}^{10} f(j)$$

$$\frac{20^2(21)^2}{4} - \frac{10^2(11)^2}{4}$$

$$\frac{20(21)}{2} - \frac{10(11)}{2} =$$

$$44,100 - 3025$$

$$210 - 55 = 155$$

$$41,075$$

$$41,075 - 4(155) =$$

$$40,455$$

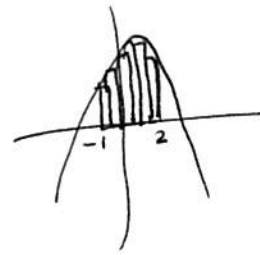
3. Estimate the area under the curve $f(x) = 8 + x - x^2$ on the interval $[-1, 2]$ using 6 rectangles and the right-endpoint rule. Sketch a graph of what you are doing.

$$\int_{-1}^2 8 + x - x^2 dx \approx \sum_{i=1}^6 f(x_i) \Delta x =$$

$$x_i = \left\{ -1, -\frac{1}{2}, 0, \frac{1}{2}, 1, \frac{3}{2}, 2 \right\}$$

$$\frac{1}{2} \left[f(-\frac{1}{2}) + f(0) + f(\frac{1}{2}) + f(1) + f(\frac{3}{2}) + f(2) \right] \approx$$

$$\frac{1}{2} [7.25 + 8 + 8.25 + 8 + 7.25 + 6] = 22.375$$



$$\frac{2 - (-1)}{6} = \frac{3}{6} = \frac{1}{2} = \Delta x$$

$$x_i = -1 + i \Delta x = -1 + \frac{i}{2}$$

$$f(x_i) = 8 + (-1 + \frac{i}{2}) - (-1 + \frac{i}{2})^2$$

$$= 8 - 1 + \frac{i}{2} - (\frac{i^2}{4} - i + 1) =$$

$$7 + \frac{i}{2} - \frac{i^2}{4} + i - 1 =$$

$$6 + \frac{3i}{2} - \frac{i^2}{4}$$

4. Find the exact value of the area under the curve on the same interval as in #3.

$$\lim_{n \rightarrow \infty} \sum_{i=1}^n \frac{3}{n} \left(6 + \frac{3i}{n} - \frac{9i^2}{n^2} \right) = \frac{3}{n} \left[\sum_{i=1}^n 6 + \frac{9}{n} \sum_{i=1}^n i - \frac{9}{n^2} \sum_{i=1}^n i^2 \right] =$$

$$\lim_{n \rightarrow \infty} \frac{3}{n} \left[6n + \frac{9}{n} \cdot \frac{n(n+1)}{2} - \frac{9}{n^2} \cdot \frac{n(n+1)(2n+1)}{6} \right] =$$

$$\lim_{n \rightarrow \infty} 3 \left[6 + \frac{9}{2n} n + \frac{9}{2n} - \frac{\frac{9}{n^2} \cdot \frac{1}{2} n^2}{\frac{1}{3}} - \frac{9 \cdot 3n}{6 \cdot n^2} - \frac{3}{n^2} \cdot \frac{1}{2} n \right]$$

$$\lim_{n \rightarrow \infty} 3 \left[6 + \frac{9}{2} + \frac{9}{2n} - 3 - \frac{9}{2n} - \frac{3}{2n^2} \right] = 3 \left[6 + \frac{9}{2} - 3 \right] = \frac{45}{2}$$

$$= 22.5$$

$$-1 + \frac{3i}{n}$$

$$8 + (-1 + \frac{3i}{n}) - (-1 + \frac{3i}{n})^2$$

$$8 - 1 + \frac{3i}{n} - (\frac{9i^2}{n^2} - \frac{6i}{n} + 1) =$$

$$8 - 1 + \frac{3i}{n} - \frac{9i^2}{n^2} + \frac{6i}{n} - 1 =$$

$$6 + \frac{9i}{n} - \frac{9i^2}{n^2}$$