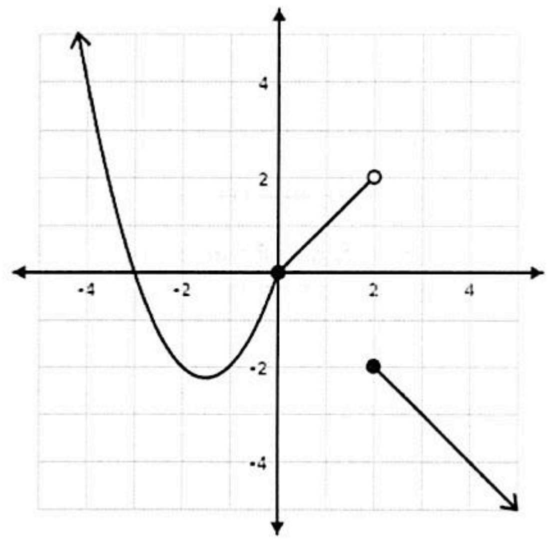
MTH 263, Quiz #2, Summer 2023 Name KEY

Instructions: Work the problems below as directed. Show all work. Clearly mark your final answers. Use exact values unless the problem specifically directs you to round. Simplify as much as possible. Partial credit is possible, but solutions without work will not receive full credit.

1. Find the limits.



i. $\lim\limits_{x\to 0^+} f(x) = 0$

ii. $\lim\limits_{x\to 0^-} f(x) = 0$

iii. $\lim\limits_{x\to 0} f(x) = 0$

iv. $\lim\limits_{x\to 2^-} f(x) = 2$

v. $\lim\limits_{x\to 2^+} f(x) = -2$

vi. $\lim\limits_{x\to 2} f(x)$ DNE

vii. Is the graph continuous at $x = 0$? yes

viii. Is the graph continuous at $x = 2$? no

ix. If the graph is not continuous at $x = 0$ or $x = 2$, describe the type of discontinuity.

at $x=2$, it is a jump discontinuity

2. Using the formal $\varepsilon - \delta$ definition of the limit, prove that $\lim\limits_{x\to 3} 4x - 5 = 7$.

$|4x-5-7|<\varepsilon$

$|4x-12| = 4|x-3| < \varepsilon$

$|x-3| < \frac{\varepsilon}{4}$

if $|x-3|<\delta$ and suppose that $\delta = \varepsilon/4$, then

$|x-3| < \varepsilon/4 \to 4|x-3|<\varepsilon \to |4x-12|<\varepsilon \to$

$|4x-5-7|<\varepsilon$. Therefore $\lim\limits_{x\to 3} 4x-5$ is equal to 7.

3. Use the formal definition of the derivative to find a formula for the derivative $f'(x)$ for the function $f(x) = 2x^2 + 6x - 9$. Then use that formula to find the slope of the tangent line at the point $x = 1$, and then write the equation of the tangent line at that point.

$$f'(x) = \lim_{h\to 0} \frac{2(x+h)^2+6(x+h)-9-(2x^2+6x-9)}{h} = \lim_{h\to 0} \frac{\cancel{2x^2}+4xh+2h^2+\cancel{6x}+6h\cancel{-9}\cancel{-2x^2}\cancel{-6x}\cancel{+9}}{h}$$

$$= \lim_{h\to 0} \frac{4xh+2h^2+6h}{h} = \lim_{h\to 0} \frac{\cancel{h}(4x+2h+6)}{\cancel{h}} = \lim_{h\to 0} 4x+2h+6 = 4x+6$$

$f'(1) = 4(1)+6 = 10$ $f(1) = 2+6-9 = -1$ $(1,-1) \to$ $y+1 = 10(x-1)$