MTH 263, Quiz #13, Summer 2023 Name KEY

Instructions: Work the problems below as directed. Show all work. Clearly mark your final answers. Use exact values unless the problem specifically directs you to round. Simplify as much as possible. Partial credit is possible, but solutions without work will not receive full credit.

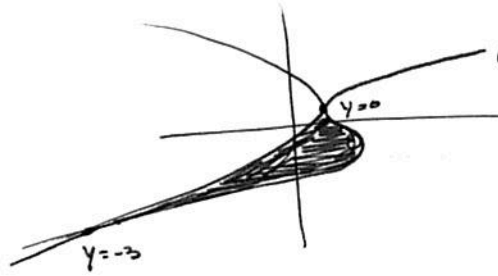
1. Find the area between the curves $x = y^3 + 2y^2 + 1$, and $x = 1 - y^2$.

$y^3 + 2y^2 + \cancel{1} = \cancel{1} - y^2$

$y^3 + 3y^2 = 0$

$y^2(y+3) = 0$

$y = 0,\ y = -3$



$\frac{1}{4}y^4 + y^3 \Big|_{-3}^{0} =$

$0 - \left[\frac{1}{4}(-3)^4 + (-3)^3\right] =$

$-\left[\frac{81}{4} - 27\right] = \frac{27}{4}$

$\int_{-3}^{0} y^3 + 2y^2 + 1 - (1 - y^2)\,dy$

$= \int_{-3}^{0} y^3 + 3y^2\,dy$

2. Find the area between the curves $y = x^3$ and $y=$ $x^2 - 2x$ on the interval [-1,2].

$x^3 = x^2 - 2x$

$x^3 - x^2 + 2x = 0$

$x(x^2 - x + 2) = 0$

$x = 0$

$\int_{-1}^{0} x^2 - 2x - x^3\,dx + \int_{0}^{2} x^3 - (x^2 - 2x)\,dx = \left[\frac{1}{3}x^3 - x^2 - \frac{1}{4}x^4\right]_{-1}^{0} + \left[\frac{1}{4}x^4 - \frac{1}{3}x^3 + x^2\right]_{0}^{2} =$

$\frac{19}{12} + 16/3 = 83/12$

3. Sketch the area bounded by $y = 2x^2, x = 0, x = 4, y = 0$. Use the washer method to find the volume of the solid rotated around the x-axis.

$\pi \int_{0}^{4} (2x^2)^2\,dx = \pi \int_{0}^{4} 4x^4\,dx =$

$\pi \left[\frac{4}{5}x^5\right|_{0}^{4} = \pi \frac{4}{5} 4^5 = \frac{4096\pi}{5}$