6/15/2023

Hyperbolic Trig Functions
Review for Exam #1

In the online textbook, you may find definitions for hyperbolic trig functions at the end of chapter 1 (last section maybe), and the calculus parts in Chapter 6.9 (at the end of the calc 1 section of the book).

We are also mostly going to ignore inverse hyperbolic trig functions.

Definitions

$$\sinh(x) = \frac{(e^x - e^{-x})}{2}$$

$$\cosh(x) = \frac{e^x + e^{-x}}{2}$$

Compare:

$$\sin(x) = \frac{\left(e^{ix} - e^{-ix}\right)}{2i}$$

$$\cos(x) = \frac{e^{ix} + e^{-ix}}{2}$$

Regular trig identity:

$$\sin^2\theta + \cos^2\theta = 1$$

Hyperbolic trig identity:

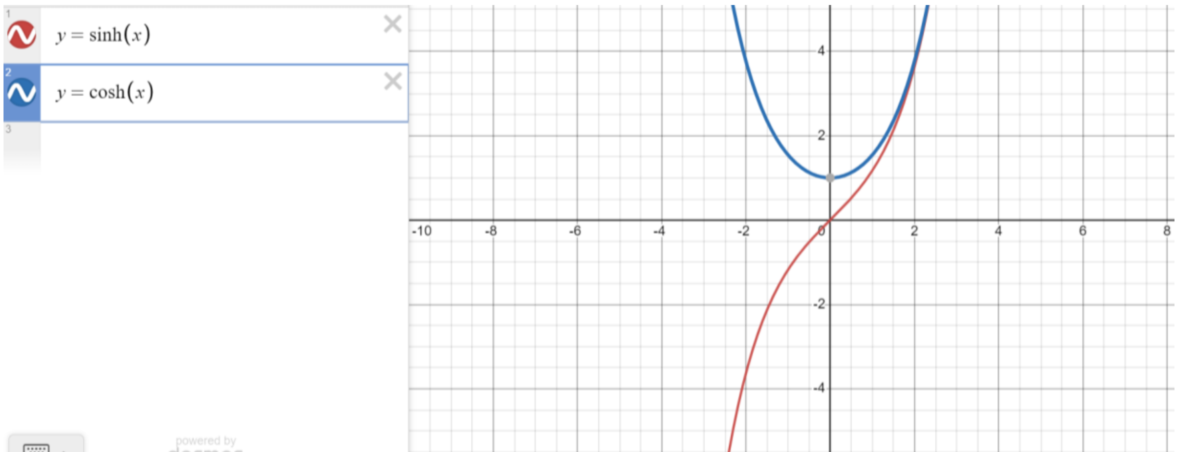
$$\cosh^2 x - \sinh^2 x = 1$$

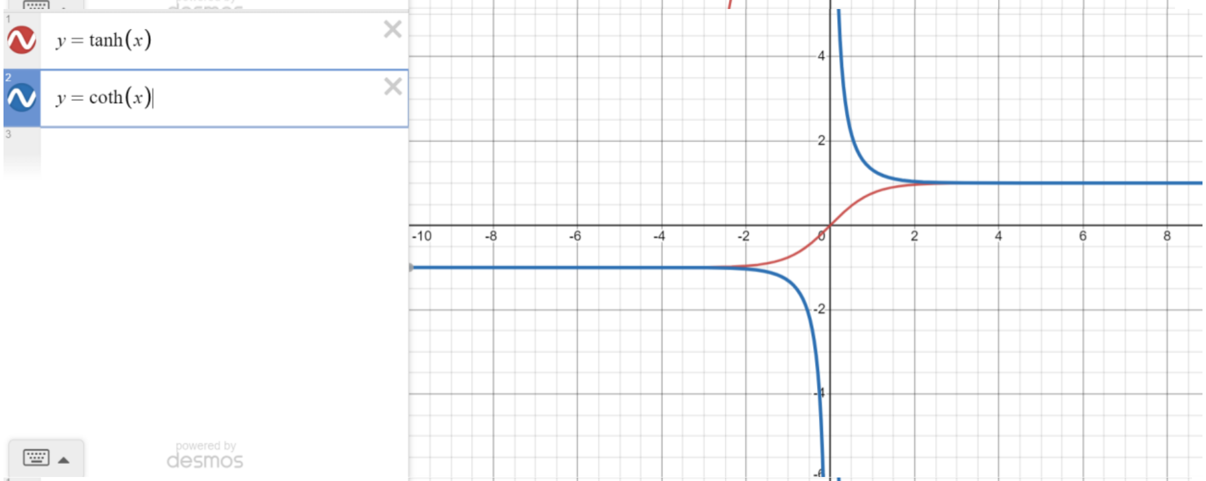
$$\tanh(x) = \frac{\sinh(x)}{\cosh(x)} = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

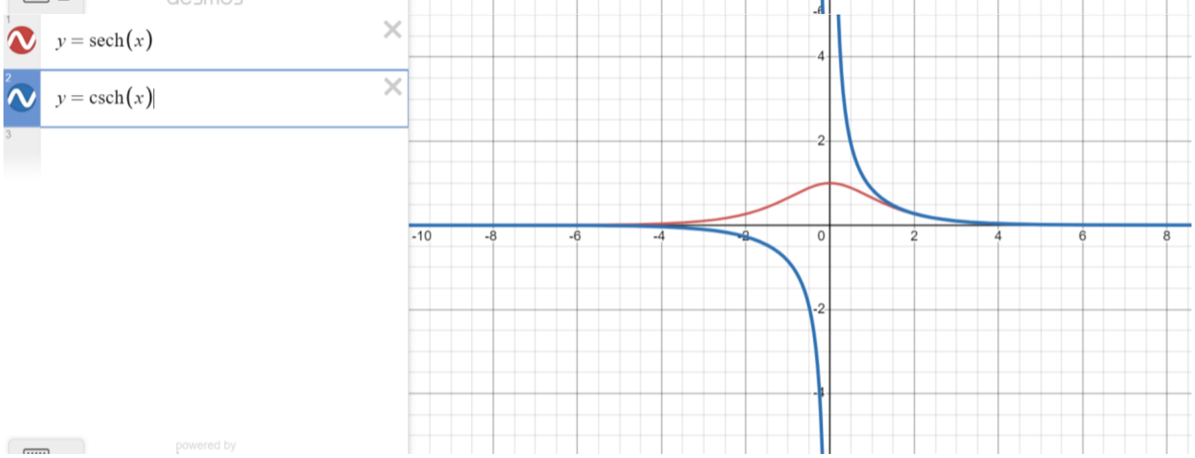
$$\coth(x) = \frac{\cosh(x)}{\sinh(x)} = \frac{e^x + e^{-x}}{e^x - e^{-x}}$$

$$\operatorname{sech}(x) = \frac{1}{\cosh(x)} = \frac{2}{e^x + e^{-x}}$$

$$\operatorname{csch}(x) = \frac{1}{\sinh(x)} = \frac{2}{e^x - e^{-x}}$$







Derivative of Hyperbolic trig

1. For hyperbolic sine and hyperbolic cosine, if you are asked to prove the derivative, then go back to the exponential definitions of the functions.
2. For all the other hyperbolic trig functions, only go back to hyperbolic sine and hyperbolic cosine (don't go back to the exponentials).

$$\frac{d}{dx}[\sinh(x)] = \frac{d}{dx}\left[\frac{e^x - e^{-x}}{2}\right] = \frac{1}{2}\frac{d}{dx}[e^x - e^{-x}] = \frac{1}{2}[e^x + e^{-x}] = \frac{e^x + e^{-x}}{2} = \cosh(x)$$

$$\frac{d}{dx}[\cosh x] = \sinh(x)$$

This is a sign difference.

$$\frac{d}{dx}[\tanh x] = \frac{d}{dx}\left[\frac{\sinh x}{\cosh x}\right] = \frac{\cosh x \cdot \cosh x - \sinh x \cdot \sinh x}{\cosh^2 x} = \frac{\cosh^2 x - \sinh^2 x}{\cosh^2 x}$$

$$= \frac{1}{\cosh^2 x} = \operatorname{sech}^2 x$$

$$\frac{d}{dx}[\coth x] = -\operatorname{csch}^2 x$$

$$\frac{d}{dx}[\operatorname{sech} x] = -\operatorname{sech} x \tanh x$$

This is a sign difference.

$$\frac{d}{dx}[\operatorname{csch} x] = -\operatorname{csch} x \coth x$$

In the original trig functions, all the “co” functions have negative derivatives (cosine, cotangent, cosecant).
But in the hyperbolic world, it’s all the reciprocal functions (cotangent, secant, cosecant).

You might be asked to prove an identity.
If you can do it without going back to the exponential definitions, that’s usually easier.