MTH 263, Homework #4, Summer 2023 Name ______________________________

Instructions: Write your work up neatly and attach to this page. Record your final answers (only) directly on this page if they are short; if too long indicate which page of the work the answer is on and mark it clearly. Use exact values unless specifically asked to round.

1. Verify the identity.
   a. $e^x = \sinh x + \cosh x$
   b. $\sinh 2x = 2\sinh x \cosh x$
   c. $\tanh^2 x + \text{sech}^2 x = 1$

2. Find the derivative of the function.
   a. $f(x) = \sinh 3x$
   b. $y = \ln\left(\tanh \frac{x}{2}\right)$
   c. $f(t) = \arctan(\sinh t)$
   d. $y = \tanh(3x^2 - 1)$
   e. $g(x) = x\cosh x - \sinh x$

3. Find the equation of the tangent line to the function $y = e^{\cosh x}$ at $(1,1)$.

4. Find the rate of change of the distance between the origin and a moving point on the graph of $y = x^2 + 1$ if $\frac{dx}{dt} = 2\frac{cm}{s}$.

5. The radius r of a sphere is increasing at a rate of $3\frac{in}{s}$. Find the rate of change of the volume when $r = 9$, and $r = 36$ inches.

6. All edges of a cube are expanding at a rate of $6\frac{cm}{s}$. How fast is the volume and surface area changing when each edge is $2\ cm$ and again at $10\ cm$?

7. A conical tank (with vertex down) is $10\ ft$ across the top and $12\ ft$ deep. If water is flowing into the tank at a rate of $10\frac{ft^3}{min}$, find the rate of change of the depth of the water when the water is $8\ ft$ deep.

8. A ladder $25\ ft$ long is leaning against the wall of a house. The base of the ladder is pulled away from the wall at $2\frac{ft}{s}$. How fast is the top of the ladder coming down the wall when the base is $7ft, 15ft,$ and $24\ ft$?

9. An airplane flies at an altitude of 5 miles toward a point directly over an observer. The speed of the plan is $600\ mph$. Find the rates of change of the angle of elevation when the angle is $\theta = 30°, \theta = 60°$ and $\theta = 75°$.

10. Find any critical points of the function.
    a. $g(x) = x^4 - 4x^2$
    b. $g(t) = 2t\ln t$
    c. $f(x) = \frac{4x}{x^2+1}$
    d. $h(x) = 4x^2(3^x)$

11. Locate the absolute extrema of the function on the interval.
    a. $h(x) = -x^2 + 3x - 5, [-2,-1]$
    c. $f(x) = \sqrt[3]{x}, [-1,1]$

b. $g(x) = \frac{x^2}{x^2+3}, [-1,1]$

d. $q(x) = \arctan x^2, [-2,1]$

12. Determine whether the mean value theorem can be applied.

a. $y = x^2, [-2,1]$

b. $g(x)e^{-3x}, [0,2]$

c. $f(x) = \frac{x+1}{x}, [-1,2]$

13. Apply the mean value theorem to the function $f(x) = x^4 - 8x$ on the interval $[-1,1]$.

14. Locate any relative extrema and state the intervals on which the function is increasing or decreasing.

a. $f(x) = -2x^2 + 4x + 3$

b. $h(x) = 2x + \frac{1}{x}$

c. $m(x) = x \arctan x$

d. $g(x) = x^4 - 32x + 4$

e. $q(x) = \frac{x^2}{x^2-9}$

f. $y = (x-1)e^x$

15. Apply the first derivative test to classify any critical points.

a. $y = \frac{x}{2} + \cos x$

b. $f(x) = \cos^2 2x$

16. Find any inflection points.

a. $y = -x^4 + 24x^2$

b. $g(x) = \frac{x+1}{\sqrt{x}}$

c. $n(x) = x - \ln x$

d. $f(x) = x\sqrt{9-x}$

e. $h(x) = e^{-\frac{3}{x}}$

f. $p(x) = \arctan x^2$

17. Use the second derivative test to classify the critical points.

a. $f(x) = (x-5)^2$

b. $h(x) = x + \frac{4}{x}$

c. $q(x) = x^2 e^{-x}$

d. $g(x) = x^4 - 4x^3 + 2$

e. $y = \frac{x}{\ln x}$

f. $s(x) = 8x(4^{-x})$

18. Find the differential of the function.

a. $y = 3x^2 - 4$

b. $y = x \cos x$

c. $y = x\sqrt{1-x}$

d. $y = x \arcsin x$

19. Use differentials to approximate the value.

a. $\sqrt{99.4}$

b. $\sqrt[3]{28}$