MTH 263, Homework #3, Summer 2023 Name KEY

Instructions: Write your work up neatly and attach to this page. Record your final answers (only) directly on this page if they are short; if too long indicate which page of the work the answer is on and mark it clearly. Use exact values unless specifically asked to round.

1. Find the derivative of the function.
   a. $y = \frac{3}{4}e^x + 2\cos x$
   b. $g(x) = \frac{\sin x}{e^x}$
   c. $g(t) = \sqrt[4]{t} + 6\csc t$
   d. $h(x) = 2\sin x \cos x$
   e. $y = \sqrt{3}\,x + 2\cos x$
   f. $h(x) = \sqrt{x}\sin x$
   g. $y = \frac{\sec x}{x}$
   h. $y = 2x\sin x + x^2e^x$

   pg 2

2. Find the second derivative of the function.
   a. $y = \tan x$
   b. $y = x\cos x + \cot x$

   pg 2

3. Find the derivative of the function.
   a. $f(x) = (4x-1)^3$
   b. $g(x) = \frac{x}{\sqrt{x^2+1}}$
   c. $y = \sqrt{\frac{1}{t^2-2}}$ $= (t^2-2)^{-1/2}$
   d. $h(x) = [(x^2+3)^5 + x]^2$
   e. $f(s) = e^{-s}\ln s$
   f. $q(x) = 4^x$
   g. $y = log_3(x)$
   h. $y = \sin(\cos x)$
   i. $y = \sqrt[3]{6x^2+1}$
   j. $y = \frac{1}{2}x^2\sqrt{16-x^2}$
   k. $g(t) = \left(\frac{3t^2-1}{2t+5}\right)^3$
   l. $f(\theta) = \tan^2 5\theta$
   m. $y = e^{\sqrt{x}}$
   n. $s(t) = t^2 2^t$
   o. $h(t) = \ln\left(\frac{x}{x^2+1}\right)$ $= \ln x - \ln(x^2+1)$

   pg 2-3

4. Find $\frac{dy}{dx}$ implicitly.
   a. $x^2 - y^2 = 25$
   b. $\sin x + 2\cos 2y = 1$
   c. $\ln(xy) + 5x = 30$
      $\ln x + \ln y + 5x = 30$
   d. $x^2y + y^2x = -3$
   e. $\cot y = x - y$
   f. $\tan(x+y) = x$

   pg 3

5. Find an equation of the tangent line to the graph $x^2 + xy + y^2 = 4$ at the point (2,0). pg 4

6. Find $\frac{d^2y}{dx^2}$ for $1 - xy = x - y$. pg 4

7. Use logarithmic differentiation to find $\frac{dy}{dx}$.
   a. $y = \frac{x(x-1)^{3/2}}{\sqrt{x+1}}$
   b. $y = (1+x)^{1/x}$
   c. $y = x^{2/x}$
   d. $y = x^{\ln x}$

   pg 4

8. Find the derivative of the function.
   a. $y = \arctan e^x$
   b. $f(x) = e^{2x}\operatorname{arcsec} 3x$
   c. $g(x) = \frac{\arccos x}{x+1}$

   pg 5

MTH 263 Homework #3 Key

a. $y' = \frac{3}{4}e^x - 2\sin x$

b. $g'(x) = \dfrac{\cos x \cdot e^x - e^x \sin x}{e^{2x}} = \dfrac{\cos x - \sin x}{e^x}$

c. $g'(t) = \frac{1}{4}t^{-3/4} - 6\csc t \cot t = \dfrac{1}{4\sqrt[4]{t^3}} - 6\csc t \cot t$

d. $h'(x) = 2\cos x \cos x + 2\sin x(-\sin x) = 2\cos^2 x - 2\sin^2 x$

e. $y' = \sqrt{x} - 2\sin x$

f. $h'(x) = \frac{1}{2}x^{-1/2}\sin x + \sqrt{x}\cos x = \dfrac{\sin x}{2\sqrt{x}} + \sqrt{x}\cos x$

g. $y' = \dfrac{\sec x \tan x \cdot x - \sec x}{x^2} = \dfrac{\sec x(x\tan x - 1)}{x^2}$

h. $y' = 2\sin x + 2x\cos x + 2xe^x + x^2e^x$

2a. $y' = \sec^2 x$

$y'' = 2\sec x \cdot \sec x \tan x = 2\sec^2 x \tan x$

b. $y' = \cos x - x\sin x - \csc^2 x$

$y'' = -\sin x - \sin x - x\cos x - 2\csc x \csc x \cot x$

$= -2\sin x - x\cos x - 2\csc^2 x \cot x$

3a. $f'(x) = 3(4x-1)^2(4) = 12(4x-1)^2$

b. $g'(x) = \dfrac{1\sqrt{x^2+1} - \frac{1}{2}(x^2+1)^{-1/2} \cdot 2x}{(x^2+1)} = \dfrac{\sqrt{x^2+1} - \frac{x}{\sqrt{x^2+1}}}{x^2+1} \cdot \dfrac{\sqrt{x^2+1}}{\sqrt{x^2+1}} = \dfrac{x^2+1-x}{(x^2+1)^{3/2}}$

c. $y = (t^2-2)^{-1/2}$

$y' = -\frac{1}{2}(t^2-2)^{-3/2}(2t) = \dfrac{-t}{(t^2-2)^{3/2}}$

d. $h'(x) = 2[(x^2+3)^5 + x][5(x^2+3)^4 \cdot 2x + 1]$

e. $f'(s) = -e^{-s}\ln s + \dfrac{e^{-s}}{s}$

f. $g'(x) = \ln 4 \cdot 4^x$

g. $y' = \dfrac{1}{(\ln 3)x}$

3h. $y' = \cos(\cos x) \cdot (-\sin x)$

i. $y' = \frac{1}{3}(6x^2+1)^{-2/3}\,(12x)$

j. $y' = x\sqrt{16-x^2} + \frac{1}{2}x^2 \cdot \frac{1}{2}(16-x^2)^{-1/2}(-2x)$

k. $g'(t) = 3\left(\frac{3t^2-1}{2t+5}\right)^2 \left[\frac{6t(2t+5)+2(3t^2-1)}{(2t+5)^2}\right]$

l. $f'(\theta) = 2\tan 5\theta \sec^2 5\theta \cdot 5$

n. $y' = e^{\sqrt{x}} \cdot \frac{1}{2}x^{-1/2}$

1. $s'(t) = 2t\,2^t + t^2 \cdot 2^t \cdot \ln 2$

). $h'(t) = \frac{1}{x} - \frac{2x}{x^2+1}$ $\qquad h(t) = \ln x - \ln(x^2+1)$

4a. $2x - 2y\,y' = 0$

$2x = 2y\,y'$

$y' = \frac{2x}{2y} = \frac{x}{y}$

b. $\cos x - 2\sin 2y \cdot 2y' = 0$

$\cos x = 4\sin 2y \cdot y'$

$y = \frac{\cos x}{4\sin 2y}$

c. $\frac{1}{x} + \frac{1}{y} \cdot y' + 5 = 0$

$\frac{1}{y} \cdot y' = -5 - \frac{1}{x}$

$y' = -5y - \frac{y}{x}$

d. $2xy + x^2y' + 2yy'x + y^2x = 0$

$x^2y' + 2xyy' = -2xy - y^2x$

$y' = \frac{-2xy - y^2x}{x^2+2xy}$

e. $-\csc^2 y \cdot y' = 1 - y'$

$y' - \csc^2 y \cdot y' = 1$

$y' = \frac{1}{1-\csc^2 y}$

f. $\sec^2(x+y)(1+y') = x$

$\sec^2(x+y) + \sec^2(x+y)\,y' = x$

$\sec^2(x+y)\,y' = x - \sec^2(x+y)$

$y' = \frac{x - \sec^2(x+y)}{\sec^2(x+y)}$

5. $2x + y + xy' + 2yy' = 0$

$$y' = \frac{-2x-y}{x+2y}$$

$$y'\Big|_{(2,0)} = \frac{-2(2)-0}{2+2(0)} = \frac{-4}{2} = -2 \qquad y = -2(x-2)$$

6. $-xy' - y = 1 - y'$

$$-xy' + y' = 1 + y$$

$$y' = \frac{1+y}{1-x}$$

$$y'' = \frac{y'(1-x) - (-1)(1+y)}{(1-x)^2} = \frac{\frac{1+y}{\cancel{1-x}}\cancel{(1-x)} + (1+y)}{(1-x)^2} = \frac{2+2y}{(1-y)^2}$$

7a. $\ln y = \ln x + \frac{3}{2}\ln(x-1) - \frac{1}{2}\ln(x+1)$

$$\frac{y'}{y} = \frac{1}{x} + \frac{3}{2(x-1)} - \frac{1}{2(x+1)}$$

$$y' = \left[\frac{1}{x} + \frac{3}{2(x-1)} - \frac{1}{2(x+1)}\right]y = \left[\frac{1}{x} + \frac{3}{2(x-1)} - \frac{1}{2(x+1)}\right]\frac{x(x-1)^{3/2}}{\sqrt{x+1}}$$

b. $\ln y = \frac{1}{x}\ln(1+x)$

$$\frac{y'}{y} = -x^{-2}\ln(1+x) + \frac{1}{x}\cdot\frac{1}{x+1}$$

$$y' = y\left[\frac{-1}{x^2}\ln(1+x) + \frac{1}{x(x+1)}\right] = (1+x)^{1/x}\left[-\frac{\ln(1+x)}{x^2} + \frac{1}{x(x+1)}\right]$$

c. $\ln y = \frac{2}{x}\cdot\ln x$

$$\frac{y'}{y} = -2x^{-2}\ln x + \frac{2}{x}\cdot\frac{1}{x}$$

$$y' = y\left[\frac{-2\ln x}{x^2} + \frac{2}{x^2}\right] = x^{2/x}\left[\frac{-2\ln x + 2}{x^2}\right]$$

d. $\ln y = \ln x \cdot \ln x = (\ln x)^2$

$$\frac{y'}{y} = 2\ln x \cdot \frac{1}{x}$$

$$y' = y\left[2\ln x \cdot \frac{1}{x}\right] = x^{\ln x}\left[\frac{2\ln x}{x}\right]$$

8a. $y' = \frac{1}{1+(e^x)^2} \cdot e^x = \frac{e^x}{1+e^{2x}}$

b. $f'(x) = 2e^{2x} \operatorname{arcsec} 3x + e^{2x} \cdot \frac{\cancel{3}}{\cancel{3}x\sqrt{9x^2-1}}$

$= 2e^{2x} \operatorname{arcsec} 3x + \frac{e^{2x}}{x\sqrt{9x^2-1}}$

c. $g'(x) = \frac{\frac{-1}{\sqrt{1-x^2}} \cdot (x+1) - 1 \cdot \arccos x}{(x+1)^2} = \frac{\frac{-(x+1)}{\sqrt{1-x^2}} - \arccos x}{(x+1)^2} =$

$\frac{-(x+1) - \arccos x \cdot \sqrt{1-x^2}}{(x+1)^2\sqrt{1-x^2}}$