MTH 263, Homework #1, Summer 2023 Name ____

Instructions: Write your work up neatly and attach to this page. Record your final answers (only) directly on this page if they are short; if too long indicate which page of the work the answer is on and mark it clearly. Use exact values unless specifically asked to round.

1. Numerically estimate the limits.

   a. $\lim_{x \to 2} \frac{x-2}{x^2-4}$

   b. $\lim_{x \to 0} \frac{\sin x}{x}$

   c. $\lim_{x \to -5} \frac{\sqrt{4-x}-3}{x+5}$

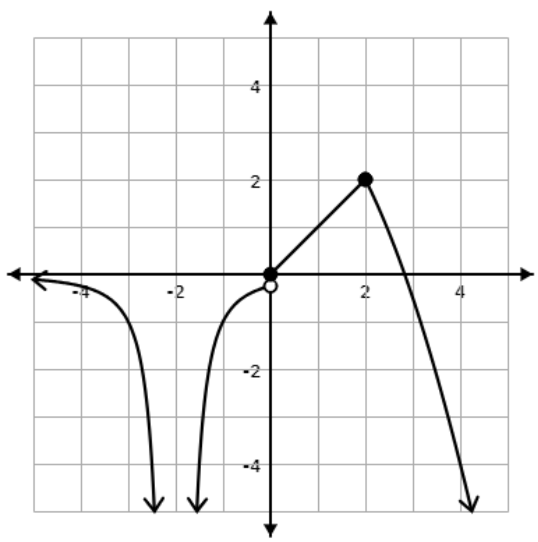
   d. $\lim_{x \to 1} \frac{x^4-1}{x^6-1}$

2. Use a graph to determine the value of the limit, if it exists.

   a. $\lim_{x \to 2} \frac{|x-2|}{x-2}$

   b. $\lim_{x \to 0} \cos\left(\frac{1}{x}\right)$

   c. $f(x) = \begin{cases} -\frac{1}{(x+2)^2}, & x < 0 \\ x, & 0 \le x \le 2 \\ -\frac{1}{2}x^2 + 4, & x > 2 \end{cases}$

   i. $\lim_{x \to 0^-} f(x)$

   ii. $\lim_{x \to 0^+} f(x)$

   iii. $\lim_{x \to 0} f(x)$

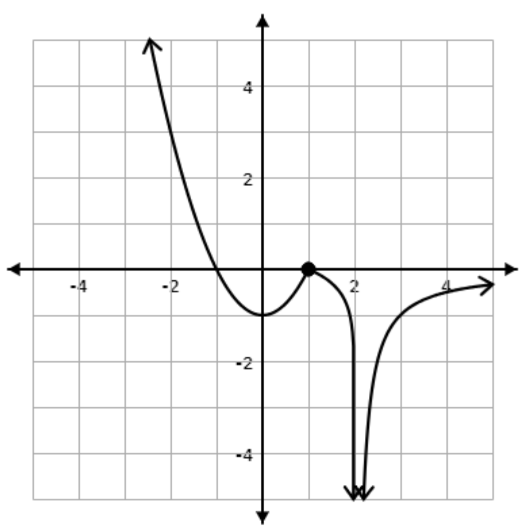
   iv. $\lim_{x \to 2^-} f(x)$

   v. $\lim_{x \to 2^+} f(x)$

   vi. $\lim_{x \to 2} f(x)$

   vii. $\lim_{x \to -2} f(x)$

   d. $f(x) = \begin{cases} x^2 - 1, & x < 1 \\ \ln(2-x), & 1 \le x < 2 \\ -\frac{1}{x-2}, & x > 2 \end{cases}$

   i. $\lim_{x \to 1^-} f(x)$

   ii. $\lim_{x \to 1^+} f(x)$

   iii. $\lim_{x \to 1} f(x)$

   iv. $\lim_{x \to 2^-} f(x)$

   v. $\lim_{x \to 2^+} f(x)$

   vi. $\lim_{x \to 2} f(x)$

   e. $\lim_{x \to 1} \sqrt[3]{x} \ln |x-2|$

3. Consider $f(x) = \begin{cases} \sin(x), & x < 0 \\ 1 - \cos(x), & 0 \le x \le \pi \\ \cos(x), & x > \pi \end{cases}$. Are there any values for c for which $\lim_{x \to c} f(x)$ does not exist?

4. Find the limit L: $\lim_{x \to 4}(4 - \frac{x}{2})$. Then find $\delta > 0$ such that $|f(x) - L| < 0.01$ whenever $0 < |x - 4| < \delta$.

5. Find the limit L. Then use the $\varepsilon - \delta$ definition of the limit to prove it.

a. $\lim_{x \to -3}(2x + 5)$

b. $\lim_{x \to 1}(x^2 + 1)$

c. $\lim_{x \to 1}\left(\frac{2}{3}x + 9\right)$

d. $\lim_{x \to 3}|x - 3|$

6. Find the limit algebraically using properties of limits.

a. $\lim_{x \to 1}(3x^3 - 4x^2 + 3)$

b. $\lim_{x \to 0} \sec 2x$

c. $\lim_{x \to 1} \ln\left(\frac{x}{e^x}\right)$

d. $\lim_{x \to 1} \frac{x^3 - x}{x - 1}$

e. $\lim_{x \to 0} \frac{e^{2x} - 1}{e^x - 1}$

f. $\lim_{x \to 0} \frac{\sin^2 x}{x}$

g. $\lim_{x \to 0} \frac{\sin 2x}{\sin 3x}$

h. $\lim_{x \to 1} \frac{2x - 3}{x + 5}$

i. $\lim_{x \to 0} e^{-x} \sin \pi x$

j. $\lim_{x \to 0} \frac{-x^2 + 3x}{x}$

k. $\lim_{x \to 2} \frac{x^3 - 8}{x - 2}$

l. $\lim_{x \to 0} \frac{\sqrt{3 + x} - \sqrt{3}}{x}$

m. $\lim_{x \to 0} \frac{1 - e^{-x}}{e^x - 1}$

n. $\lim_{x \to 0} \frac{\sin 3x}{x}$

7. Use the squeeze theorem to find the limits.

a. $\lim_{x \to 0} x \cos x$

b. $\lim_{x \to 0} x \cos \frac{1}{x}$

8. Find the limit if it exists.

a. $\lim_{x \to 2} f(x), f(x) = \begin{cases} x^2 - 4x + 6, & x < 2 \\ -x^2 + 4x - 2, & x \ge 2 \end{cases}$

b. $\lim_{x \to 6^-} \ln(6 - x)$

c. $\lim_{x \to 1^+} f(x), f(x) = \begin{cases} x, & x < 1 \\ 1 - x, & x \ge 1 \end{cases}$

9. Find any points of discontinuity. Is the discontinuity removable or not?

a. $f(x) = \frac{3}{x - 2}$

b. $f(x) = \frac{|x - 3|}{x - 3}$

c. $f(x) = \frac{x}{x^2 + 1}$

d. $f(x) = \begin{cases} -2x, & x \le 2 \\ x^2 - 4x + 1, & x > 2 \end{cases}$

10. Find the value of a that makes the function continuous. $g(x) = \begin{cases} \frac{4\sin x}{x}, x < 0 \\ a - 2x, x \geq 0 \end{cases}$

11. Find the vertical asymptotes, if they exist.

a. $f(x) = \frac{4}{(x-2)^3}$

b. $f(x) = \frac{-3x^3+12x+9}{x^4-3x^3-x+3}$

12. Find the limit.

a. $\lim_{x\to-\infty} \frac{x}{\sqrt{x^2-x}}$

c. $\lim_{x\to\infty} \frac{x^2+3}{2x^2-1}$

b. $\lim_{x\to\infty} \frac{8}{4-10^{-\frac{x}{2}}}$