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Properties of Exponents
Scientific Notation

Properties of exponents

Multiplying two numbers with exponents with the same base

$$a^n a^m = a^{n+m}$$

$$2^3 2^4 = 2^7$$

$$(2 \times 2 \times 2)(2 \times 2 \times 2 \times 2) = 2^7$$

Dividing two numbers with exponents using the same base

$$\frac{a^n}{a^m} = a^{n-m}$$

$$\frac{2^8}{2^5} = 2^3$$

$$\frac{2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2}{2 \times 2 \times 2 \times 2 \times 2} = 2^3$$

Raising exponents to exponents

$$(a^n)^m = a^{nm}$$

$$(2^3)^4 = (2 \times 2 \times 2)(2 \times 2 \times 2)(2 \times 2 \times 2)(2 \times 2 \times 2) = 2^{12}$$

Negative exponents

$$a^{-n} = \frac{1}{a^n}$$

$$2^{-3} = \frac{1}{2 \times 2 \times 2} = \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = \frac{1}{2^3}$$

If a number has a negative exponent in one part of a fraction, you can move it to the other part and change the sign of the exponent.

Any number raised to the zero power is 1.

$$a^0 = 1$$

Example.

$$\frac{2^3}{2^3} = 2^{3-3} = 2^0 = 1$$

In multiplication, 1 is the identity, and so when you cancel you get a 1, not a zero.

Example. Simplify.

$$\frac{2^3 3^5}{2^2 3^8} = \left(\frac{2^3}{2^2}\right)\left(\frac{3^5}{3^8}\right) = (2^{3-2})(3^{5-8}) = 2(3^{-3}) = \frac{2}{3^3}$$

$$\frac{2^3 3^5}{2^2 3^8} = \left(\frac{2^3}{2^2}\right)\left(\frac{3^5}{3^8}\right) = \frac{(2^{3-2})}{3^{8-5}} = \frac{2}{(3^3)} = \frac{2}{3^3}$$

It is often desirable to have only positive exponents, but that's not necessarily universal.

Example.

$$\frac{a^5 b^{-2}}{a^2 b^3} = \left(\frac{a^5}{a^2}\right)\left(\frac{b^{-2}}{b^3}\right) = (a^{5-2})\left(\frac{1}{b^3 b^2}\right) = \frac{a^3}{b^5}$$

$$\frac{a^5 b^{-2}}{a^2 b^3} = \left(\frac{a^5}{a^2}\right)\left(\frac{b^{-2}}{b^3}\right) = (a^{5-2})\left(\frac{1}{b^{3-(-2)}}\right) = \frac{a^3}{b^5}$$

$$\frac{a^5 b^{-2}}{a^2 b^3} = \left(\frac{a^5}{a^2}\right)\left(\frac{b^{-2}}{b^3}\right) = (a^{5-2})(b^{-2-3}) = a^3 b^{-5} = \frac{a^3}{b^5}$$

When the bases of the two exponents are different, but the exponent is the same

$$a^n b^n = (ab)^n$$

$$2^3 3^3 = (2 \times 2 \times 2)(3 \times 3 \times 3) = (2 \times 3)(2 \times 3)(2 \times 3) = 6 \times 6 \times 6 = 6^3$$

This is the only time you can multiply the bases together in an exponent.

Scientific Notation is based on place holder values

1,000,000 = one million, 10^6
Ones place = 10^0=1
Tens place = 10^1, recall that 70 = 7x10, 76 is 7x10+6x1
Hundreds place =10^2,10x10=100
Thousands place = 10^3= 10x10x10=1000
Ten-thousands place = 10^4
Hundred-thousands place = 10^5
Millions place = 10^6
Etc.
After the decimal, we divide by power of 10
0.000001 = millionths place = $\frac{1}{1{,}000{,}000} = \frac{1}{10^6} = 10^{-6}$

0.1 tenths position = $\frac{1}{10} = 10^{-1}$
0.01 hundredths position = $\frac{1}{100} = 10^{-2}$

0.001 thousandths position = $\frac{1}{1000} = 10^{-3}$
0.0001 ten-thousandths position = $\frac{1}{10{,}000} = 10^{-4}$
Etc.

Scientific notation is a way of writing large or small numbers in terms of powers of 10.
Large numbers will have positive exponents, and small numbers will have negative exponents.

The national debt is around 31 trillion dollars = 31,000,000,000,000

Any non-zero digits in the number should be written as a number between 1 and 9.99999999…. times a power of 10.

3|1,000,000,000,000 = 3.1x10^{13}

1,000,000 = 1×10^6

6 = 6x10^0

76 = 7.6x10^1

320 = 3.2x10^2

104,570 = 1.0457x10^5

5.97219$\times$ 10^{24} kg = 5,972,190,000,000,000,000,000,000 = approximately 6 septillion kg (mass of the Earth)

Smaller numbers (less than 1) use negative exponents.

0.004 = 4x10^{-3} = $\frac{4}{1000}$

0.000561 = 5.61x10^{-4}

Mass of a proton $1.67 \times 10^{-27} kg$
0.000 000 000 000 000 000 000 000 001 67 kg

Addition and subtraction on scientific notation is generally not done in scientific notation. Go back to standard decimal notation and then do the operation, and then convert back.

$$3.24 \times 10^4 + 5.66 \times 10^2 = 32{,}400 + 566 = 32{,}966 = 3.2966 \times 10^4$$

We don't have to do that for multiplication, division or exponents

Multiplication

$$(3.24 \times 10^4)(5.66 \times 10^2) = (3.24 \times 5.66)(10^4 \times 10^2) = 18.3384 \times 10^6$$

This number is not in scientific notation anymore, the number being multiplied by the power of 10 is not between 1 and 9.99999999….

$$18.3384 \times 10^6 = (1.83384 \times 10^1) \times 10^6 = 1.83384 \times 10^7$$

Division

$$\frac{1.85 \times 10^3}{7.25 \times 10^6} = \left(\frac{1.85}{7.25}\right)\left(\frac{10^3}{10^6}\right) = 0.25517 \ldots \times 10^{-3}$$

$$0.25517 \ldots \times 10^{-3} = (2.5517 \ldots \times 10^{-1}) \times 10^{-3} = 2.5517 \ldots \times 10^{-4}$$

Combinations

$$\frac{(5.7 \times 10^3)(2.1 \times 10^{-11})}{3.4 \times 10^9} = \left(5.7 \times \frac{2.1}{3.4}\right)\left(10^3 \times \frac{10^{-11}}{10^9}\right) = 3.520 \ldots \times 10^{3+(-11)-9} = 3.520 \ldots \times 10^{-15}$$

Exponents with scientific notation

$$(2.1 \times 10^4)^3 = (2.1 \times 10^4)(2.1 \times 10^4)(2.1 \times 10^4) = (2.1)^3(10^4)^3 = 9.261 \times 10^{12}$$

Significant digits

The significant digits in a number are the expressed digits preceding any final zeros.

$$5.7 \times 10^3 = 5700$$

This number contains 2 significant digits.

Final zeros for large numbers, and preceding zeros for small numbers don’t count.

0.004 = 4×10^{-3} has only 1 significant digit.

0.0040 = 4.0$\times 10^{-3}$ has 2 significant digits

Large numbers:

$5.70 \times 10^3 = 57\bar{0}0$ is saying that I have three significant digits.

Operation on scientific notation:

If you are working with numbers in scientific notation, and significant digits are noted, then the final answer after the operation, should have no more significant digits than the smallest number of significant digits in the original numbers.

For example, suppose you are working with two numbers, one that has 2 significant digits and one that has three significant digits, round your final answer to two significant digits.

Our addition example from earlier:

$$3.24 \times 10^4 + 5.66 \times 10^2 = 32{,}400 + 566 = 32{,}966 = 3.2966 \times 10^4$$

Before I didn't round it, but I should… both have 3 significant digits, and so my final answer should too

$$3.29|66 \times 10^4 \rightarrow 3.30 \times 10^4$$

Multiplication example:

$$(3.24 \times 10^4)(5.66 \times 10^2) = 1.83|384 \times 10^7 \rightarrow 1.83 \times 10^7$$

Exponents with scientific notation

$$(2.1 \times 10^4)^3 = (2.1 \times 10^4)(2.1 \times 10^4)(2.1 \times 10^4) = (2.1)^3(10^4)^3 = 9.2|61 \times 10^{12} \rightarrow 9.3 \times 10^{12}$$