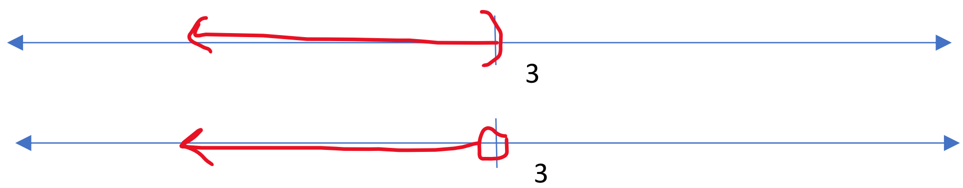
7/22/2023

Graphing on a number line
Inequalities

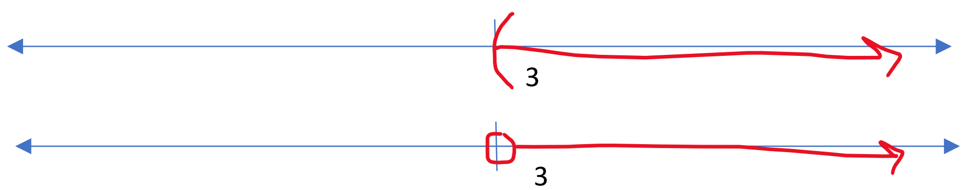
Inequalities: visualizing on a number line and mathematical notation: 1) interval notation, 2) set notation

Simple one-sided inequality: $x < 3$
Meaning: x can be any value to the left of 3 on the number line, but it can't be 3



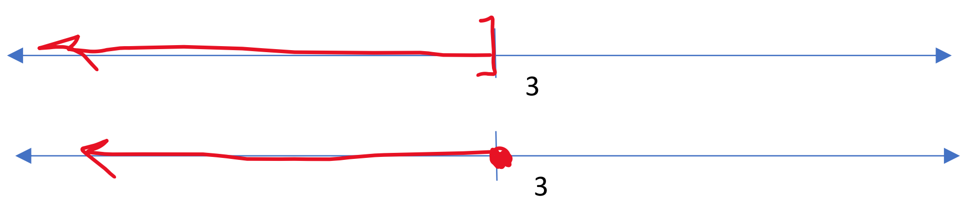
The open parentheses indicates that the 3 is not included (matches the < notation in our original inequality), and opens to the left to indicate which side of 3 is included (to the left means less than). The open circle means the same thing, but the arrow is the only thing that tells you the direction of the inequality.

What about $x > 3$



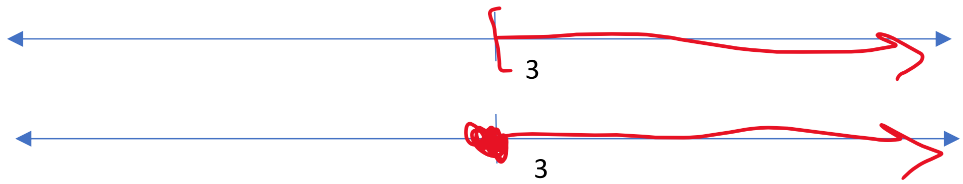
Open parenthesis (opening to the right) means x is greater than 3, and rightward arrow reinforces this. Open circle at 3 plus the rightward arrow means the same thing: values of x strictly greater than 3.

What about $x \leq 3$



The difference here is that the square bracket means that the 3 is included (matches the equality sign in the $\leq$). It open to the left because it is a less than. The closed (filled-in) circles means the same in combination with the leftward arrow.

What about $x \geq 3$



The square bracket indicates that 3 is included in the values of x, and then opens to the right with the arrow to show direction being greater than 3. The closed circle also says 3 is included, and the arrow to the right is greater than.

Interval notation

An interval is a pair of values. The first one is the left end of the interval, and the second one is the right end of the interval. If the interval has no end, then use an infinity symbol. The interval is bounded by either parentheses (if the end is not included) or a square bracket (if the end is included).

Inequality	Interval Notation
$x < 3$	$(-\infty, 3)$
$x > 3$	$(3, \infty)$
$x \leq 3$	$(-\infty, 3]$
$x \geq 3$	$[3, \infty)$

Always use parentheses (never square brackets) on infinity (it's not a number).

The values in the interval notation always have to appear in the order (left-to-right) that they appear on the number line.

Set Notation

Set Builder Notation

Sets {}

Set Builder Notation: $\{x|conditional\ statement(s)\}$

"x such that" the condition is true

$x < 3$ in set builder notation is: $\{x|x < 3\}$

Read as "x such that x is less than 3"

$x > 3$ in set builder notation is $\{x|x > 3\}$

$x \leq 3$ in set builder notation is $\{x|x \leq 3\}$

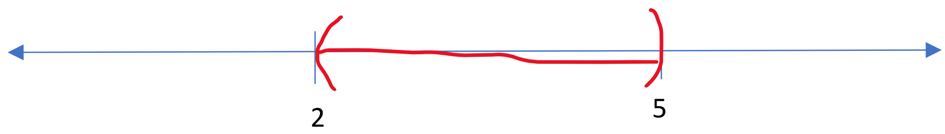
$x \geq 3$ in set builder notation is $\{x|x \geq 3\}$

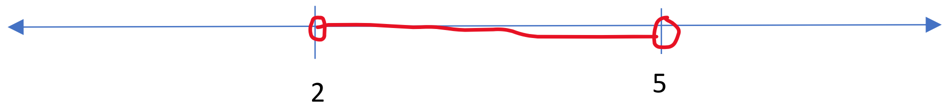
Double Inequalities

Example.

$$2 < x < 5$$

Double inequalities: there is a number on both ends, the inequality has to point in the same direction on either side of x (though they can have different equality components), generally these can be read as "x is in between 2 and 5".





Less thans are most common for these double inequalities, and then the numbers should appear in the same order as they appear on the number line (smallest to the left, and largest to the right).

Example:

$$2 > x > 5$$

This expression is meaningless. These inequalities have to be transitive, and 2 is not bigger than 5.

$$2 < x < 5$$

Is okay because 2 is less than 5.

In interval notation:

$$(2,5)$$

This means any value between 2 and 5 (not including 2 and 5)

Set Builder notation: $\{x|2 < x < 5\}$

Double Inequality	Interval Notation
$2 < x < 5$	$(2,5)$
$2 \leq x < 5$	$[2,5)$
$2 < x \leq 5$	$(2,5]$
$2 \leq x \leq 5$	$[2,5]$

When the interval notation has two parentheses, these are referred to as open intervals
When the interval notation has two square brackets, these are referred to as closed intervals
When the brackets/parentheses are different on either end, these are referred to as either half-open or half-closed intervals.

Solve Inequalities

Linear inequalities solve the same way as regular equalities (equations), but there is an extra consideration if you use the multiplication property with a negative number.

Example.

$$3x + 1 < 7$$

Use addition property to get x by itself

$$3x + 1 - 1 < 7 - 1$$
$$3x < 6$$

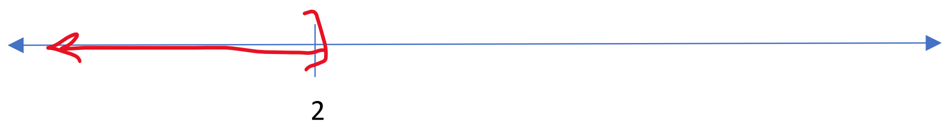
Use the multiplication property to divide by 3

$$\frac{3x}{3} < \frac{6}{3}$$

$$x < 2$$

My solution is a simplified inequality (single-sided since I started out with only one inequality)

Write the solution on the number line, in interval notation and in set-builder notation.



Interval: $(-\infty, 2)$, Set Builder Notation: $\{x|x < 2\}$

Example.

$$3(x-2)+1 \geq 2(4x+3)-7$$

$$3x-6+1 \geq 8x+6-7$$
$$3x-5 \geq 8x-1$$

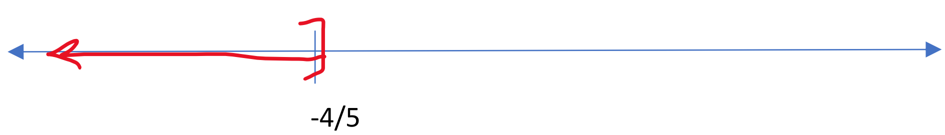
$$3x-5+5 \geq 8x-1+5$$
$$3x \geq 8x+4$$
$$3x-8x \geq 8x-8x+4$$

$$-5x \geq 4$$

Now we need x alone… but if we use the multiplication rule, then we will divide by a negative number. When we do that, we have to flip the direction of the inequality.

$$-\frac{5x}{-5} \leq \frac{4}{-5}$$

$$x \leq -\frac{4}{5}$$



Interval Notation: $\left(-\infty, -\frac{4}{5}\right]$, set builder notation: $\left\{x \middle| x \leq -\frac{4}{5}\right\}$.

Another way to get this same answer without flipping the inequality: keep the coefficient of x positive

$$3x-5 \geq 8x-1$$
$$3x-3x-5 \geq 8x-3x-1$$

$$-5 \geq 5x - 1$$
$$-5 + 1 \geq 5x - 1 + 1$$

$$-4 \geq 5x$$
$$-\frac{4}{5} \geq \frac{5x}{5}$$

$$-\frac{4}{5} \geq x$$

Compare:
$-\frac{4}{5} \geq x$ vs. $x \leq -\frac{4}{5}$

Double inequalities (solving them)

$$5 < 3x - 7 \leq 14$$

Add 7 to all three parts of this inequality to get x term by itself

$$5 + 7 < 3x - 7 + 7 \leq 14 + 7$$

$$12 < 3x \leq 21$$

Divide all three parts by 3 to get x alone

$$\frac{12}{3} < \frac{3x}{x} \leq \frac{21}{3}$$

$$4 < x \leq 7$$

Interval notation: $(4,7]$, set notation: $\{x|4 < x \leq 7\}$

This problem is equivalent to:

$$5 < 3x - 7 \leq 14$$

$$5 < 3x - 7 \; AND \; 3x - 7 \leq 14$$

If you do that, then you would have to put the pieces back together at the end.