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Converting between fractions, decimals and percentages

Ratios, rates, proportions, probabilities

Fractions, Decimals and Percentages are all rational numbers, because they can all be represented as a fraction (ratio of integers), as long as the decimal is terminating or repeating.

We have covered some of these conversions (and we'll review those), but we will add a complete set of methods.

Decimals to percentages:

Multiply the decimal by 100 to convert to a percentage (then tack on the percentage sign).

Alternatively: you can move the decimal place two places to the **right**.

$$0.321 \rightarrow 0.321 \times 100\% = 32.1\%$$

Percentages to decimals:

Divide by 100 to convert a percentage to a decimal (drop the percent sign).

Alternatively: you can move the decimal place two places to the **left**.

$$28.4\% \rightarrow \frac{24.8}{100} = 0.248 = \frac{24.8 \times 10}{100 \times 10} = \frac{248}{1000} = 0.248$$

Decimals/Percentages to Fractions (Terminating decimals):

If you start with a percentage, typically it's best to start converting to a decimal.

If you have a decimal, then use your knowledge of place value to express the decimal as a fraction.

$$0.321 = \frac{321}{1000}$$

Once it's in the form of a fraction, then simplify to lowest terms.

$$\frac{248}{1000} = \frac{124}{500} = \frac{62}{250} = \frac{31}{125}$$

Terminating decimals will always have only factors of 2 and 5 in the denominator (terminating decimals have a power of 10 in the denominator).

What about a repeating decimal?

$$0.\overline{543} = 0.543543543543 \dots = \frac{543}{999} = \frac{181}{333}$$

$$0.555555 \dots = 0.\overline{5} = \frac{5}{9}$$

$$0.444444 \dots = 0.\overline{4} = \frac{4}{9}$$

$$0.3131313131 \dots = 0.\overline{31} = \frac{31}{99}$$

$$0.7474747474 \dots = 0.\overline{74} = \frac{74}{99}$$

Some simple fractions have long repetition strings:

$$\frac{1}{7} = 0.142857142857 \dots = 0.\overline{142857} = \frac{142857}{999999} = \frac{15873}{111111} = \frac{5291}{37037} = \frac{481}{3367} = \frac{37}{259} = \frac{1}{7}$$

Digits in front and then repeating...

$$0.52666666 \dots = 0.52\overline{6} = 0.52 + 0.00\overline{6} = \frac{52}{100} + \frac{6}{900} = \frac{52 \times 9}{100 \times 9} + \frac{6}{900} = \frac{468}{900} + \frac{6}{900} = \frac{474}{900}$$

$$= \frac{237}{450} = \frac{79}{150}$$

Separate the repeating part from the non-repeating part. Write the non-repeating part as a whole number over a power of 10, then the repeating part is written with the same trick as above, but multiply the denominator by the number of leading 0's after the split (with the non-repeating part).

Combine the fractions and reduce.

$$4.3333 \dots = 4\frac{1}{3} = 4.\overline{3} = 4 + 0.\overline{3} = 4 + \frac{3}{9} = 4 + \frac{1}{3} = 4\frac{1}{3} = \frac{13}{3}$$

The process is basically the same here, but that the whole number has a denominator of 1 so the finding a common denominator step is easier (can be skipped).

Converting fractions to decimals:

Divide.

$$\frac{3}{11} \Rightarrow \begin{array}{r} 2727 \dots \\ 11 \overline{) 3.000} \\ \underline{22} \\ 80 \\ \underline{77} \\ 30 \end{array}$$

$$\frac{3}{11} = 0.272727 \dots = 0.\overline{27}$$

To convert fractions to percentages: go through the decimal process first, and then convert to a percentage from there.

$$\frac{3}{11} = 0.27272727 \dots = 27.272727 \dots \% = 27.\overline{27}\% = 27\frac{3}{11}\%$$

Fraction	Decimal	Percentage
$\frac{4}{7}$	$0.\overline{571428}$	$57.\overline{142857} = 57\frac{1}{7}\%$
$\frac{592}{1000} = \frac{296}{500} = \frac{148}{250} = \frac{74}{125}$	0.592	59.2%
$\frac{32}{100} = \frac{16}{50} = \frac{8}{25}$	0.32	32%

Applications of ratios:

Fractions are ratios: ratios of integers, but ratios don't have to be integers in the way that fractions do require.

Ratios can be expressed in different ways

Sometimes ratios are expressed as fractions: "3 to 4" is written as $\frac{3}{4}$

Sometimes ratios are expressed with colons: "3 to 4" is written as 3:4

Colon is read as "to"

Proportion problems are built on ratios.

We can write these problems in terms of fractions (which we will do to solve them), but they can also be written in the colon notation.

$$\frac{3}{4} = \frac{x}{2}$$

A proportion problem is one ratio that is known, and one ratio that is only partially known set equal to each other. The goal is to find the missing part so that the two ratios are equivalent.

The colon notation:

$$3:4 :: x:2$$

A rose is to flowers as what is to trees?

$$rose:flowers :: x:trees$$

A proportion is basically a double comparison that we are trying to match.

$$\frac{3}{4} = \frac{x}{2}$$

The trick for solving a proportion problem (once we set up the equation) is to cross multiply across the equal sign.

$$3(2) = 4(x)$$

$$6 = 4x$$

$$\frac{6}{4} = x$$

$$x = 1.5$$

$$\frac{3}{4} = ? \frac{1.5}{2}$$

Example.

One brand of microwave popcorn has 120 calories per serving. A whole bag of this popcorn has 3.5 servings. How many calories are in a whole bag of this microwave popcorn?

Set up proportion problems by comparing like to like.

$$\frac{1}{120} = \frac{\text{servings}}{\text{calories}} = \frac{3.5}{x}$$

$$(\text{left side is servings}) \frac{1}{3.5} = \frac{120}{x} (\text{right side is calories})$$

Both of these setups will solve the problem, and the answers are equivalent as long as you are comparing like to like (in the same order).

$$\frac{1}{120} = \frac{3.5}{x}$$

$$x = 420$$

$$\frac{1}{3.5} = \frac{120}{x}$$

$$x = 420$$

Unit conversion is a very common application of this method.

$$1 \text{ mile} = 1.6 \text{ km}$$

How many kilometers is 60 miles?

$$\frac{1 \text{ mi}}{1.6 \text{ km}} = \frac{60}{x}$$

$$x = 60(1.6) = 96 \text{ km}$$

What number is 52% of 400?

$$\frac{52}{100} = \frac{x}{400} = \frac{\text{part}}{\text{base}}$$

$$100x = 52(400)$$
$$x = 208$$

50 is what percent of 350?

$$\frac{50}{350} = \frac{x}{100}$$

Rates : are always ratios

100 km/hr

$$\frac{100 \text{ km}}{1 \text{ hr}} = \frac{x}{5}$$

How far can I travel in 5 hours at this rate?

Probabilities are also ratios: part/whole

There is a 50% chance of rain: $50/100 = \frac{1}{2}$

We'll do scientific notation next week.