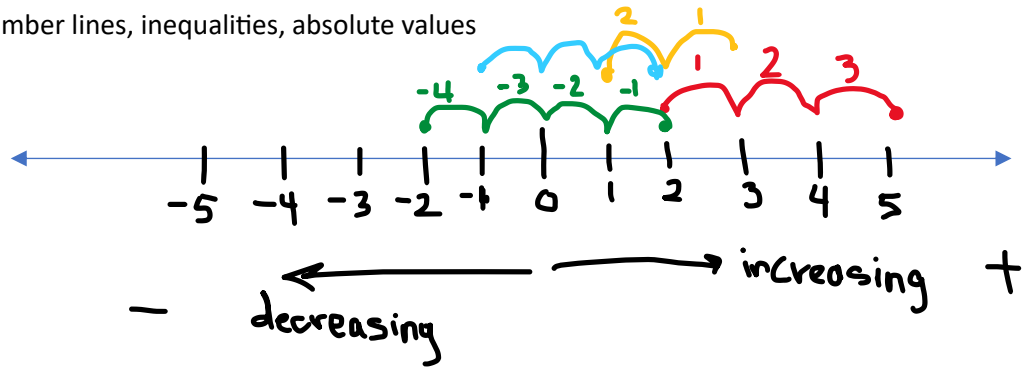
6/9/2023

Operations on Integers
Properties of numbers: associative, commutative, identity, subtraction/negatives, distributive, inverses, etc.
Number lines, inequalities, absolute values



2+3 = 5
Start at 1… jump up three more units (to the right), and where you land is the sum.
3-2 = 1
Starting at 3 and then jumping back (to the left) 2 units and stopping at 1.

When we add a negative number, instead of jumping to the right, we jump to the left. When you add a positive number you are increasing the value (rightward), but when you add a negative number, you move to the left because you are decreasing the value.

2+(-4) = -2
Start at 2. Jump leftward 4 units (because it's -4), and I stop at -2.

-1-(-3) = 2
Starting at -1. If we were adding, we'd go left by 3, but since we are subtracting, we go the opposite direction, which rightward… 3 steps right of -1 is 2 that is where we stop.

Adding a negative is equivalent to subtracting the positive version of the number.
Subtracting a negative is equivalent to adding the positive version of the number.

2+(-4)=-2
2-4 = -2

-1-(-3)=2
-1+3=2

Algorithm for subtract/adding numbers of different signs:

Subtract the smaller number from the larger number, and then take the sign of the larger number.

2-4 = 2+(-4)
Subtract 4-2 : 2

Then since the larger number is negative, the answer will be negative: -2

-1+3:
Subtract 3-1 = 2, but the larger number is 3 which means the answer is still positive because 3 is positive.

Consider, what if I wanted to find 43-172?

This is equivalent to 43+(-172). The "bigger" number is 172 and it's negative.
Subtract
172
-43
129
Then the answer is given the negative sign from that bigger number.
43-172= -129.

If both numbers that are being added are the same sign, then add the numbers normally, and the answer will have the common sign.

-31-84 = (-31)+(-84)= -115
Add 31 + 84, and then answer will be negative
31
+84
115

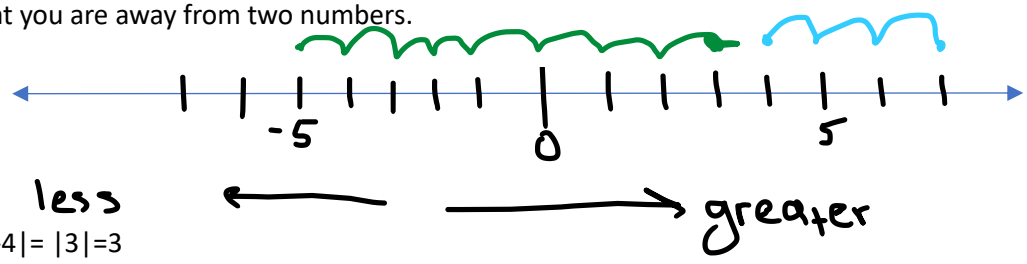
28-(-33) = 28+33=61

Absolute values
|x|: if the number inside is positive, then do nothing, but if the number inside is negative, make it positive.

|4|=4
|-4|=4

-|-4|= -(4) = -4

Think of absolute value like a distance. If you are on the number line, it's the number of units (jumps) that you are away from two numbers.



|7-4|= |3|=3
|-5-(3)|=|-8|=8

When you take the absolute value of a single number, you are just asking, how far away (how many jumps) are you away from zero.

Sometimes we refer to the “size” of a number as its magnitude, this refers to the absolute value.
When we talk about a number being “greater” than another number, we usually mean rightward on the number line.

Multiplication and Division.
For positive numbers: multiplication is built on the idea of repeated addition.
5x3 = 5+5+5 =15
(-5)x3 = (-5)+(-5)+(-5) = -15
if I multiply two positive numbers, I get a positive number
If I multiply one negative and one positive number, I get a negative number
If I multiply two negative numbers, I get a positive number

5x(-3)= 0-5-5-5= -15
(-5)x(-3)=0-(-5)-(-5)-(-5)= 15

Different signs, the answer is negative
If they are the same sign, the answer is positive.
(negative signs in pairs)

Division works the same way:
If you divide a positive number by a positive number, the answer is positive
If you divide a positive number by a negative number, the answer is negative
If you divide a negative number by a positive number, the answer is negative
If you divide a negative number by a negative number, the answer is positive

(-11)x(-5)= 55
(-11)x(5)=(11)x(-5)= -55

72/8 = 9
-72/8 = -9
72/(-8) = -9
(-72)/(-8) = 9

Exponents and roots:
Exponents are defined by multiplication:

$$2^2 = 2 \times 2 = 4$$
$$2^3 = 2 \times 2 \times 2 = 8$$

$$(-2)^2 = (-2) \times (-2) = 4$$
$$(-2)^3 = (-2) \times (-2) \times (-2) = 4 \times (-2) = -8$$
$$(-2)^4 = (-2) \times (-2) \times (-2) \times (-2) = 4 \times 4 = 16$$

If you are raising a negative number to an even power, there are an even number of negatives, that cancel in pairs, and give you a positive number.

If you are raising a negative number to an odd power, there is an odd number of negatives, and that means the answer is negative.

There is a difference between $(-2)^2$ and -2^2.

Order of operations:
Parentheses, exponents, multiplication/division, addition/subtraction

$$(-2)^2 = (-2)(-2) = 4$$
$$-2^2 = -2(2) = -4$$

Properties of numbers/Rules

If you add two numbers together, it does not matter which order you add them in: $a + b = b + a$
Ex. 2+3=3+2=5
Commutative property

The order that you multiply numbers in doesn't matter: $a \times b = b \times a$
Ex. 2(3)=3(2)= 6

Subtraction and division don't work this way.

$$4 - 2 \neq 2 - 4$$
$$\frac{15}{3} \neq \frac{3}{15}$$

Associative rule:
If you are adding (or multiplying) three numbers, you can add (or multiply) any pair together first, and then do the third one.

$$(a + b) + c = a + (b + c)$$
$$(ab)c = a(bc)$$

In addition, 0 is the identity.

$$a + 0 = a$$

3+0=3
In multiplication, 1 is the identity:

$$a(1) = a$$

3(1)=3

Inverses:
In addition, -a is the opposite of a and if I add them, I get 0.

$$a + (-a) = 0, a - a = 0$$

If multiplication/division:

$$\frac{a}{a} = 1$$

5/5 =1, 9/9 = 1, -11/-11 =1
3+(-3)=0, (-7)-(-7)= 0

$$|7-9|+(-3)^2-4\times 5\div 2-(-12)$$

Parentheses:

$$|-2|+(-3)^2-4\times 5\div 2+12$$

$$2+(-3)^2-4\times 5\div 2+12$$

Exponents:

$$2+9-4\times 5\div 2+12$$

Multiplication/Division:

$$2+9-20\div 2+12$$
$$2+9-10+12$$

Addition/Subtraction:

$$11-10+12$$
$$1+12$$
$$13$$