

Instructions: Work the problems below as directed. Show all work. Clearly mark your final answers. Use exact values unless the problem specifically directs you to round. Simplify as much as possible. Partial credit is possible, but solutions without work will not receive full credit.

1. Determine if the infinite series converge or diverge. Explain your reasoning, and which test you used to determine it.

a. $\sum_{k=2}^{\infty} \frac{\sqrt{k}}{\ln^{10}(k)}$

$$\int_2^{\infty} \frac{\sqrt{k}}{\ln^{10}(k)} dk \text{ diverges therefore the series diverges}$$

integral test

b. $\sum_{k=1}^{\infty} k e^{-k^2}$

$$\int_1^{\infty} k e^{-k^2} dk \text{ integral test converges}$$

$$= \frac{e^{-1}}{2} = \frac{1}{2e}$$

2. Determine if the alternating series converges conditionally, absolutely or diverges. Explain your reasoning.

$$\sum_{k=1}^{\infty} \frac{(-1)^k}{\sqrt{k}} \text{ converges conditionally}$$

$$\sum_{k=1}^{\infty} \frac{1}{\sqrt{k}} = \sum_{k=1}^{\infty} \frac{1}{k^{1/2}} \text{ by the p-test, } p < 1$$

therefore non-alternating series diverges

3. Find the minimum number of terms that would be needed to approximate each series to better than three decimal places accuracy (i.e. $R < 0.001$). If it cannot be approximated, explain why not.

$$\sum_{k=1}^{\infty} \frac{(-1)^k k}{k^4 + 1}$$

$$0.001 \geq \frac{k}{k^4 + 1}$$

$$k = 10$$