MTH 264, Quiz #8, Summer 2022 Name KEY

Instructions: Work the problems below as directed. Show all work. Clearly mark your final answers. Use exact values unless the problem specifically directs you to round. Simplify as much as possible. Partial credit is possible, but solutions without work will not receive full credit.

1. Find the limit of the sequence $a_n = \frac{1}{2}\arctan(n)$ if it exists. If it does not, state that it diverges.

$\lim_{n\to\infty} \frac{1}{2}\arctan n = \frac{1}{2}\left(\frac{\pi}{2}\right) = \pi/4$

2. Write an expression for the nth term of the sequence

$$1, x, \frac{x^2}{2}, \frac{x^3}{6}, \frac{x^4}{24}, \frac{x^5}{120}, \ldots$$

$n=0$ $n=2$

$$\boxed{\frac{x^n}{n!}}$$

3. Determine whether the series converge. And if so, to what.

a. $\sum_{n=0}^{\infty} \frac{17}{3}\left(-\frac{8}{9}\right)^n$

Since $\left(\frac{8}{9}\right) < 1$ this series converges by geometric series test

$$= \frac{17/3}{1-(-8/9)} = \frac{17/3}{1+8/9} = \frac{17/3}{17/9} = \frac{\cancel{17}}{3} \cdot \frac{9}{\cancel{17}} = \boxed{3}$$

b. $\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$

$\frac{A}{n} + \frac{B}{n+1} = A(n+1) + Bn = 1$

$A+B=0$

$A=1 \quad B=-1$

$\sum_{n=1}^{\infty}\left(\frac{1}{n} - \frac{1}{n+1}\right)$ telescoping series

$$\frac{1}{1} - \lim_{n\to\infty} \frac{1}{n+1} = \boxed{1}$$