MTH 264, Quiz #7, Summer 2022 Name KEY

Instructions: Work the problems below as directed. Show all work. Clearly mark your final answers. Use exact values unless the problem specifically directs you to round. Simplify as much as possible. Partial credit is possible, but solutions without work will not receive full credit.

1. Integrate the following improper integrals. Determine if the integrals converge or diverge. If they converge, find the values they converge to.

a. $\int_0^\infty e^{-x}\cos x\,dx = \lim_{b\to\infty}\int_0^b e^{-x}\cos x\,dx = \lim_{b\to\infty}\frac{1}{2}\left[e^{-x}\sin x - e^{-x}\cos x\right]_0^b$

$= \lim_{b\to\infty}\frac{1}{2}\left[e^{-b}\sin b - e^{-b}\cos b - e^0\sin 0 + e^0\cos 0\right] = \frac{1}{2}$

converges

b. $\int_0^1 x\ln x\,dx = \lim_{a\to 0}\int_a^1 x\ln x\,dx = \lim_{a\to 0}\frac{x^2}{2}\ln x - \frac{x^2}{4}\Big|_a^1 =$

$u=\ln x \quad dv = x\,dx$
$du=\frac{1}{x}dx \quad v=\frac{1}{2}x^2$

$\lim_{a\to 0}\frac{(1)^2}{2}\ln(1) - \frac{(1)^2}{4} - \frac{a^2}{2}\ln(a) + \frac{a^2}{4} = -\frac{1}{4}$ converges

$\lim_{a\to 0}\frac{a^2\ln(a)}{2} = \lim_{a\to 0}\frac{\ln(a)}{2a^{-2}} = \lim_{a\to 0}\frac{\frac{1}{a}}{-4a^{-3}} = \lim_{a\to 0}\frac{1}{a}\cdot\frac{a^3}{-4} = \lim_{a\to 0}\frac{a^2}{-4} = 0$

2. Consider the integral $\int_1^4 \ln x\,dx$. This integral cannot be calculated exactly by any method we know so far, so we need to approximate the integral numerically. We need an error to be within $E \le 0.0000001 = 10^{-7}$ of the exact value. Calculate the number of subintervals (n) needed for Simpson's Rule to achieve this value. You may use the formula $E \le \frac{(b-a)^5}{180n^4}\left[\max\left|f^{IV}(x)\right|\right]$, for $a \le x \le b$. (10 pts.)

$y = \ln x$
$y' = \frac{1}{x} = x^{-1}$
$y'' = -x^{-2}$
$y''' = 2x^{-3}$
$y^{IV} = -6x^{-4} = \frac{-6}{x^4}$

max at smallest x.

$10^{-7} = E \approx \frac{(4-1)^5}{180n^4}\left[\frac{\pm 6}{1}\right] \Rightarrow 10^{-7} = \frac{35.6}{180n^4}$

$n^4 = \frac{35.6\cdot 10^7}{180} = 81000000$

$n \geq 94.8683$

$n = 95$