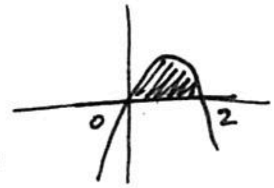
MTH 264, Quiz #3, Summer 2022 Name KEY

Instructions: Work the problems below as directed. Show all work. Clearly mark your final answers. Use exact values unless the problem specifically directs you to round. Simplify as much as possible. Partial credit is possible, but solutions without work will not receive full credit.

1. Find the centroid of the lamina of uniform density bounded by the graphs $y = 2x - x^2, y = 0$.

$$M = \rho \int_0^2 2x - x^2 \, dx = \rho \left[x^2 - \tfrac{1}{3}x^3 \right|_0^2 = \tfrac{4}{3}\rho$$



$$M_x = \frac{\rho}{2} \int_0^2 (2x - x^2)^2 \, dx = \frac{\rho}{2} \int_0^2 x^4 - 4x^3 + 4x^2 \, dx =$$

$$\frac{\rho}{2} \left[\tfrac{1}{5}x^5 - x^4 + \tfrac{4}{3}x^3 \right|_0^2 = \frac{8}{15}\rho$$

$$M_y = \rho \int_0^2 x(2x - x^2) \, dx = \rho \int_0^2 2x^2 - x^3 \, dx = \rho \left[\tfrac{2}{3}x^3 - \tfrac{1}{4}x^4 \right|_0^2 = \tfrac{4}{3}\rho$$

$$\bar{x} = \frac{M_y}{M} = \frac{4/3\,\rho}{4/3\,\rho} = 1$$

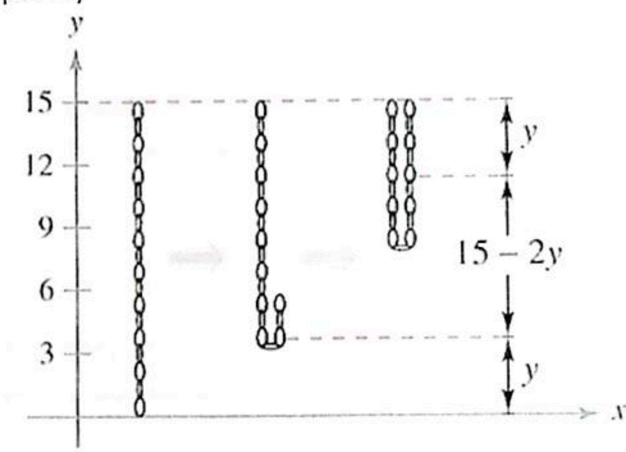
$$(\bar{x}, \bar{y}) = \left(1, \tfrac{2}{5}\right)$$

$$\bar{y} = \frac{M_x}{M} = \frac{8/15\,\rho}{4/3\,\rho} = \frac{2}{5}$$

2. Consider a 15-foot hanging chain that weighs three pounds per foot. Find the work done in lifting the chain vertically to take the bottom of the chain and raise it to the 15-foot level leaving the chain doubled and still hanging vertically. (12 points)

$$W = 3 \int_0^{7.5} (15 - 2y) \, dy =$$

$$3 \left[15y - y^2 \right|_0^{7.5} = 168.75 \text{ ft/lbs.}$$



3. Find the average value of the function $f(x) = e^{x-1}$, on the interval $[-1, ln(2) + 1]$. Round your answer to two decimal places.

$$\frac{1}{\ln 2 + 1 - (-1)} \int_{-1}^{\ln 2 + 1} e^{x-1} \, dx = \frac{1}{\ln 2 + 2} \left[e^{x-1} \right|_{-1}^{\ln 2 + 1} = \frac{(2e^2 - 1)e^{-2}}{\ln 2 + 2} \approx 0.692374$$