MTH 264, Quiz #14, Summer 2022 Name KEY

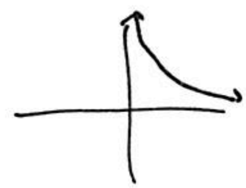
Instructions: Work the problems below as directed. Show all work. Clearly mark your final answers. Use exact values unless the problem specifically directs you to round. Simplify as much as possible. Partial credit is possible, but solutions without work will not receive full credit.

1. Convert the rectangular equation to polar form, or the polar form to rectangular form and sketch the graph. Then find the equation of the tangent line to the graph at the indicated point.

   a. $xy = 4$ (1,4) $y = \frac{4}{x}$ $\frac{dy}{dx} = -\frac{4}{x^2} \to -4$

   $r\cos\theta \cdot r\sin\theta = 4$ $y-4 = -4(x-1)$

   $r^2 = 4\sec\theta\csc\theta$



   b. $r = 5(1-2\sin\theta)$ (-5,π/2) rect. $(0,-5)$

   $r^2 = 5r - 10r\sin\theta$

   $x^2+y^2 = 5\sqrt{x^2+y^2} - 10y$

$$\frac{dy}{dx} = \frac{-10\cos\theta\cdot\sin\theta + 5(1-2\sin\theta)\cos\theta}{-10\cos\theta\cos\theta - 5(1-2\sin\theta)\sin\theta}$$

$$= \frac{-20\cos^2\theta\sin\theta + 5\cos\theta}{-10\cos^2\theta - 5 + 10\sin^2\theta} = \frac{5\cos\theta}{-5+10\sin\theta} = \frac{5}{+5} = 0$$

   $y+5 = 0(x-0)$

   $y = -5$

2. Find the area of the region: one petal of $r = 2\cos 3\theta$

$$A = \frac{1}{2}\int_{-\pi/6}^{\pi/6} (2\cos 3\theta)^2\, d\theta = \frac{1}{2}\int_{-\pi/6}^{\pi/6} 4\cos^2 3\theta\, d\theta = 2\cdot 2\int_0^{\pi/6} \frac{1}{2}(1-\cos 6\theta)\, d\theta$$

$$= 2\int_0^{\pi/6} 1-\cos 6\theta\, d\theta = 2\left[\theta - \frac{1}{6}\sin 6\theta\right]_0^{\pi/6} = 2\left[\frac{\pi}{6}\right] = \frac{\pi}{3}$$

3. Find the slope of the tangent line to the graph $r = 3 - 2\cos\theta$ when $\theta = \frac{\pi}{3}$. You may use the formula $\frac{dy}{dx} = \frac{f'(\theta)\sin\theta + f(\theta)\cos\theta}{f'(\theta)\cos\theta - f(\theta)\sin\theta}$. [Hint: to write the line, you may do it in rectangular coordinates, but you will have to convert the point on the polar graph to rectangular as well.]

$$\frac{dy}{dx} = \frac{2\sin\theta\cdot\sin\theta + (3-2\cos\theta)\cos\theta}{2\sin\theta\cos\theta - (3-2\cos\theta)\sin\theta} = \frac{2\sin^2\theta + 3\cos\theta - 2\cos^2\theta}{4\sin\theta\cos\theta - 3\sin\theta} =$$

$$\frac{2\left(\frac{\sqrt{3}}{2}\right)^2 + 3\left(\frac{1}{2}\right) - 2\left(\frac{1}{2}\right)^2}{4\left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{2}\right) - 3\left(\frac{\sqrt{3}}{2}\right)} = \frac{2\left(\frac{3}{4}\right) + \frac{3}{2} - 2\left(\frac{1}{4}\right)}{4\left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{2}\right) - \frac{3}{2}\sqrt{3}} = \frac{5/2}{\sqrt{3} - \frac{3}{2}\sqrt{3}} \times \frac{2}{2} = \frac{5}{2\sqrt{3} - 3\sqrt{3}} =$$

$$\frac{5}{-\sqrt{3}}$$