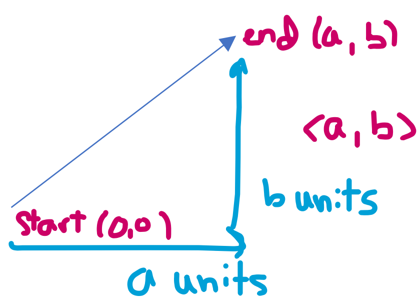
8/3/2022

Vectors
Review for Final

Previously talked about vectors as a way to contain parametric as a single object.

Vectors can be thought of as their own thing and then those properties apply to these vector functions.

A vector can be thought of as an ordered pair (in component form) that indicate the amount moved from one end of the vector to the other in each direction.



If we have two points we can find the vector between them by subtracting their corresponding components.

Vector between (1,6) and (3,4) is $\langle 3-1, 4-6\rangle = \langle 2, -2\rangle$

Another component form is using unit vectors in each direction. $2\hat{\imath} + (-2)\hat{\jmath}$
If there is a component with a k, in the z direction.

Vectors can be thought of as objects that have a magnitude and a direction.
Recall that $x^2 + y^2 = r^2$, and $\theta = \tan^{-1}\left(\frac{y}{x}\right)$
We can use these same formulas to find the magnitude and direction of 2D vectors.
Magnitude is $2^2 + (-2)^2 = r^2 \rightarrow r^2 = 8 \rightarrow r = 2\sqrt{2}$.
Direction is $\theta = \tan^{-1}\left(-\frac{2}{2}\right) = \tan^{-1}(-1) = -\frac{\pi}{4}$ … check if this is in the right quadrant?

If we want to go back to component form we use $x = r\cos\theta$, $y = r\sin\theta$

$$x = 2\sqrt{2}\cos\left(-\frac{\pi}{4}\right) = 2\sqrt{2}\left(\frac{1}{\sqrt{2}}\right) = 2$$

$$y = 2\sqrt{2}\sin\left(-\frac{\pi}{4}\right) = 2\sqrt{2}\left(-\frac{1}{\sqrt{2}}\right) = -2$$

$$< 2, -2 >$$

If we want to add vectors, we add them in component form.

Add the vectors $\vec{u} = \langle 1,4\rangle$, $\vec{v} = \langle -2,3\rangle$

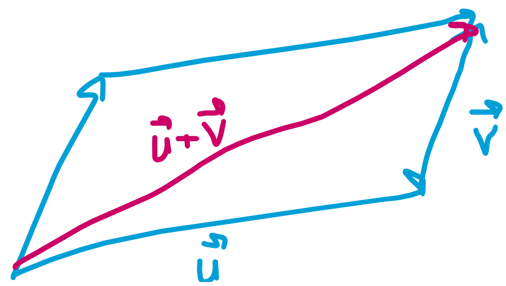
$$\vec{u} + \vec{v} = \langle -1,7 \rangle$$

Scale vectors by multiplying them by a constant.

$$3\vec{u} = \langle 3,12 \rangle$$

$$3\vec{u} - \vec{v} = \langle 3,12 \rangle - \langle -2,3 \rangle = \langle 5,9 \rangle$$

When you add vectors together, the resulting vector is the diagonal of a parallelogram with the vectors as sides.



If you are given vectors in magnitude and direction form, you need to convert to component form and then add them. Put them back in magnitude and direction form.

Suppose you have two forces acting on an object. One force is 10 lbs. acting in an angle of 45-degrees with the positive x-axis. And a second force of 15 lbs. acting at an angle of 30-degrees with the negative x-axis. What is the resulting force and direction?

$$F_1 =< 10\cos\left(\frac{\pi}{4}\right), 10\sin\left(\frac{\pi}{4}\right) > = < \frac{10}{\sqrt{2}}, \frac{10}{\sqrt{2}} >$$

$$F_2 =< 15\cos\left(\frac{5\pi}{6}\right), 15\sin\left(\frac{5\pi}{6}\right) > = < -\frac{15\sqrt{3}}{2}, \frac{15}{2} >$$

$$F_{total} =< \frac{10}{\sqrt{2}} - \frac{15\sqrt{3}}{2}, \frac{10}{\sqrt{2}} + \frac{15}{2} >$$

Find the magnitude and direction from there.

Use the angle format for your final answer that you started with.

Dot products are another way to multiply vectors. This is sometimes called a scalar product because it produces a scalar (a number).

Find the dot product of the vectors $\vec{u} = \langle 1,4 \rangle$, $\vec{v} = \langle -2,3 \rangle$

$$\vec{u} \cdot \vec{v} = u_1 v_1 + u_2 v_2 = (1)(-2) + (4)(3) = -2 + 12 = 10$$

Turns out that the dot product is related to the angle between the two vectors.

$$\cos\theta = \frac{\vec{u} \cdot \vec{v}}{\|\vec{u}\|\|\vec{v}\|} = \frac{10}{\sqrt{17}\sqrt{13}}$$

Positive dot products point to acute angles (between 0 and 90-degrees where the cosine is positive), and negative dot products point to obtuse angles (between 90 and 180-degrees where the cosine is negative).

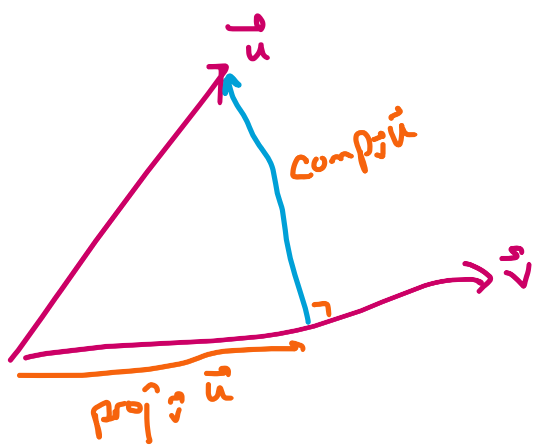
If the dot product is zero then the angle between them is exactly 90-degrees. The vectors are perpendicular. (orthogonal).

What is a vector that is orthogonal/perpendicular to my $\vec{u}$ vector?
Generally $\langle 1,4\rangle \cdot \langle a, b\rangle = 0 \rightarrow a + 4b = 0$
The vector orthogonal to $\langle a, b\rangle$ is going to be either $\langle b, -a\rangle$ or $\langle -b, a\rangle$.
Flip the components and change one sign.

$$\langle 1,4\rangle \cdot \langle 4,-1\rangle = (1)(4) + (4)(-1) = 4 - 4 = 0$$

Projections (orthogonal projections)
Orthogonal decomposition: project a vector onto another vector and then subtract the result from the original vector, and what you get is two vectors that are perpendicular to each other but which add to the original projected vector.

$$proj_{\vec{v}}(\vec{u}) = \frac{\vec{u} \cdot \vec{v}}{||\vec{v}||^2}(\vec{v})$$



$$proj_{\vec{v}}(\vec{u}) = \frac{\vec{u} \cdot \vec{v}}{||\vec{v}||^2}(\vec{v}) = \frac{10}{13}\langle -2,3\rangle = \langle -\frac{20}{13}, \frac{30}{13}\rangle$$

To find the orthogonal complement, subtract from the original u vector.

Work is the dot product of the force dotted with the direction of motion.

$$W = \vec{F} \cdot \vec{d}$$

To find the domain of a vector function (vector-valued function), find the domain of each component and then see where all of them are true at the same time.

3D parametric curve grapher https://www.geogebra.org/m/ubratnur or search for another one.

$$\|r(t)\| = \sqrt{x(t)^2 + y(t)^2 + z(t)^2}$$

The arc length for vector functions (parametric functions in vector form)

$$s = \int_a^b \|r'(t)\|\, dt = \int_a^b \sqrt{[x'(t)]^2 + [y'(t)]^2}\, dt$$

Review for the Final

About half of the exam is on the last two chapters (4&7)
Of the remaining half, about 2/3 of that is on the first two chapters (2&3), the rest on (5&6)
Review the quizzes and tests for the older material.

To request partial credit consideration… (I recommend doing this after the final has been graded).
Submit the exam and the problem number (it would be helpful if you included the value of the full points worth of the problem).