7/20/2022

Finish Taylor Series
Review for Exam #2

Example.

$$f(x) = \sinh x$$

1. Method 1: you use the definition of a Taylor series to generate the function the "long" way.
2. Method 2: go back to the definition of hyperbolic sine and use the exponential Taylor series that we derived on Monday (from the table).

Recall: $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24} + \cdots$

And recall: $\sinh x = \frac{e^x - e^{-x}}{2} = \frac{1}{2}(e^x - e^{-x})$

$$\frac{1}{2}(e^x - e^{-x}) = \frac{1}{2}\left(\sum_{n=0}^{\infty} \frac{x^n}{n!} - \sum_{n=0}^{\infty} \frac{(-1)^n x^n}{n!}\right) =$$

$$\frac{1}{2}\left[1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24} + \cdots - \left(1 - x + \frac{x^2}{2} - \frac{x^3}{6} + \frac{x^4}{24} - \cdots\right)\right] = \frac{1}{2}\left(0 + 2x + 0 + \frac{2x^3}{6} + 0 + \cdots\right) =$$

$$x + \frac{x^3}{6} + \frac{x^5}{120} + \cdots = \sum_{n=0}^{\infty} \frac{x^{2n+1}}{(2n+1)!}$$

Doing division with Taylor series

When working with Taylor series or Taylor polynomials, generally speaking use ascending order. Start with the constant or the smallest degree of your Taylor series for both the numerator and the denominator.

Suppose I want to generate a Taylor series for $f(x) = \frac{\sin x}{x+1}$, find the first three terms.

1) Find your Taylor series for any basic functions. $\text{Sin}\, x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!} = x - \frac{x^3}{6} + \frac{x^5}{120} - \frac{x^7}{5040} + \cdots$
2) Set up your division with the constant (or lowest degree) leading

$$x - x^2 + \frac{5}{6}x^3 \cdots$$

$$1+x \;\overline{\big)\, x + 0x^2 - \frac{x^3}{6} + 0x^4 + \frac{x^5}{120} \cdots}$$

$$\frac{x}{1} = x$$

$$-x - x^2$$

$$-x^2 - \frac{x^3}{6} + 0x^4 + \frac{x^5}{120} \cdots$$

$$+x^2 + x^3$$

$$\frac{5}{6}x^3 + 0x^4 + \frac{x^5}{120} \cdots$$

$$-\frac{5}{6}x^3 - \frac{5}{6}x^4$$

$$-\frac{5}{6}x^4 + \frac{x^5}{120} \cdots$$

Taylor series for $\frac{\sin x}{1+x} = x - x^2 + \frac{5}{6}x^3 - \frac{5}{6}x^4 + \cdots$

Exam #2

Covers Chapters 5 and 6 in the online textbook

- Sequences – go back and review the basics. These questions are meant to be easy, just don't forget how to do them (or that they exist).
- Series tests – mostly for convergence, but there is one test of divergence
- Two types of series which we can find exact sums for (geometric and telescoping)
- Two types of series that have error estimation methods (alternating and integral test)
- Power series – need to be able to construct them, and test for intervals of convergence
- Taylor series – need to be able to construct them, find the estimated error using our error formula, and combine them using arithmetic/deriving more complex formulas from basic series.

$$\frac{n^{2n}}{(n+1)^{2n+2}} = \frac{n^{2n}}{(n+1)^{2n}(n+1)^2} = \left(\left(\frac{n}{n+1}\right)^n\right)^2 \times \frac{1}{(n+1)^2}$$

L'Hopitals is for indeterminant forms:

$$\frac{0}{0}, \frac{\infty}{\infty}, 0 \times \infty, 1^\infty, 0^0, etc.$$