MTH 264, Homework #8, Summer 2022 Name ____

Instructions: Write your work up neatly and attach to this page. Record your final answers (only) directly on this page if they are short; if too long indicate which page of the work the answer is on and mark it clearly. Use exact values unless specifically asked to round.

1. Find the Maclaurin polynomial of the specified degree for the function. (You can use the tables provided at the back of this assignment.)
   a. $f(x)=e^{-x}, n=3$
   b. $f(x)=xe^{x}, n=4$
   c. $f(x)=\tan x, n=4$
   d. $f(x)=\sin \pi x, n=4$
   e. $f(x)=\dfrac{x}{x+1}, n=5$ (hint: divide first)

2. Find the nth Taylor polynomial centered at the given c. (You can use the tables provided at the back of this assignment.) If no n is specified, the expression is finite. Go until $f^{(n)}(x)=0$.
   a. $f(x)=\dfrac{1}{x}, n=4, c=1$
   b. $f(x)=x^2\cos x, n=3, c=\pi$
   c. $f(x)=x-x^3, c=-2$
   d. $f(x)=\sqrt[3]{x}, n=3, c=8$
   e. $f(x)=x^4-3x^2+1, c=1$

3. Using the function from problem 1a, find the Maclaurin polynomial for n=6. Then graph each polynomial $P_0, P_1, P_2, P_3, P_4, P_5, P_6$ against the graph of the original function (do one approximation at a time; you should have 7 graphs when you are finished, each one with the original function and one approximation). You can you graphing software to graph the functions, and you can sketch them from the graph.

4. Use the table in the book to find the Taylor series for the function, centered at the specified c (or c=0 if none is specified).
   a. $f(x)=e^{3x}$
   b. $f(x)=\sinh x$
   c. $f(x)=2\sin x^3$
   d. $f(x)=e^{x}\cos x$
   e. $f(x)=x^2\ln(1+x^3)$
   f. $f(x)=\ln(x^2+1)$
   g. $f(x)=\cos 4x, c=\dfrac{\pi}{2}$
   h. $f(x)=\cos^2 x$
   i. $f(x)=\dfrac{\sin x}{1+x}$
   j. $f(x)=\tan^{-1}x^3$

5. Use a Taylor series to find an expression for the (in)definite integral for the definite integrals. Use six terms of your expression to estimate the value of the integral.
   a. $\int x\cos x^3\,dx$
   b. $\int \frac{e^x-1}{x}\,dx$
   c. $\int_0^1 \sin x^4\,dx$
   d. $\int_0^{\frac{1}{2}} x^2 e^{-x^2}\,dx$

6. Use a series to find the limit.

a. $\lim_{x\to 0} \frac{x-\ln(1+x)}{x^2}$

b. $\lim_{x\to 0} \frac{1-\cos x}{1+x-e^x}$

7. Find the sum of the series. [Hint: match the series to a Taylor or Maclaurin series to evaluate it.]

a. $\sum_{n=0}^{\infty} \frac{(-1)^n x^{4n}}{n!}$

b. $\sum_{n=1}^{\infty} \frac{(-1)^{n-1} 3^n}{n 5^n}$

c. $\sum_{n=0}^{\infty} \frac{(-1)^n \pi^{2n}}{6^{2n}(2n)!}$

8. For each function below, determine the number of terms needed to calculate the error on the Taylor series centered at c, to within $E \le 0.0001$ (10^{-4}) of the value of the function at a. Calculate your estimate and compare to the true value.

a. $f(x) = e^{2x}, c = 0, a = 1$

b. $f(x) = \sqrt[3]{8+x}, c = 2, a = 2.5$

n	n!	$f^{(n)}(x)$	$f^{(n)}(c)$	$(x-c)^n$	$\frac{f^{(n)}(c)}{n!}(x-c)^n$
0					
1					
2					
3					
4					
5					
6					

$P_n(x) =$

n	n!	$f^{(n)}(x)$	$f^{(n)}(c)$	$(x-c)^n$	$\frac{f^{(n)}(c)}{n!}(x-c)^n$
0					
1					
2					
3					
4					
5					
6					

$P_n(x) =$

n	n!	$f^{(n)}(x)$	$f^{(n)}(c)$	$(x-c)^n$	$\frac{f^{(n)}(c)}{n!}(x-c)^n$
0					
1					
2					
3					
4					
5					
6					

$P_n(x) =$

n	n!	$f^{(n)}(x)$	$f^{(n)}(c)$	$(x-c)^n$	$\frac{f^{(n)}(c)}{n!}(x-c)^n$
0					
1					
2					
3					
4					
5					
6					

$P_n(x) =$

n	n!	$f^{(n)}(x)$	$f^{(n)}(c)$	$(x-c)^n$	$\frac{f^{(n)}(c)}{n!}(x-c)^n$
0					
1					
2					
3					
4					
5					
6					

$P_n(x) =$

n	n!	$f^{(n)}(x)$	$f^{(n)}(c)$	$(x-c)^n$	$\frac{f^{(n)}(c)}{n!}(x-c)^n$
0					
1					
2					
3					
4					
5					
6					

$P_n(x) =$

n	n!	$f^{(n)}(x)$	$f^{(n)}(c)$	$(x-c)^n$	$\frac{f^{(n)}(c)}{n!}(x-c)^n$
0					
1					
2					
3					
4					
5					
6					

$P_n(x) =$

n	n!	$f^{(n)}(x)$	$f^{(n)}(c)$	$(x-c)^n$	$\frac{f^{(n)}(c)}{n!}(x-c)^n$
0					
1					
2					
3					
4					
5					
6					

$P_n(x) =$

n	n!	$f^{(n)}(x)$	$f^{(n)}(c)$	$(x-c)^n$	$\frac{f^{(n)}(c)}{n!}(x-c)^n$
0					
1					
2					
3					
4					
5					
6					

$P_n(x) =$

n	n!	$f^{(n)}(x)$	$f^{(n)}(c)$	$(x-c)^n$	$\frac{f^{(n)}(c)}{n!}(x-c)^n$
0					
1					
2					
3					
4					
5					
6					

$P_n(x) =$

n	n!	$f^{(n)}(x)$	$f^{(n)}(c)$	$(x-c)^n$	$\frac{f^{(n)}(c)}{n!}(x-c)^n$
0					
1					
2					
3					
4					
5					
6					

$P_n(x) =$

n	n!	$f^{(n)}(x)$	$f^{(n)}(c)$	$(x-c)^n$	$\frac{f^{(n)}(c)}{n!}(x-c)^n$
0					
1					
2					
3					
4					
5					
6					

$P_n(x) =$