MTH 264, Homework #7, Summer 2022 Name ____

Instructions: Write your work up neatly and attach to this page. Record your final answers (only) directly on this page if they are short; if too long indicate which page of the work the answer is on and mark it clearly. Use exact values unless specifically asked to round.

1. Determine the convergence or divergence of the series. Use any test that is appropriate. State the test you use.

   a. $\sum_{n=1}^{\infty} \frac{2n}{n+1}$

   b. $\sum_{n=1}^{\infty} \frac{n7^n}{n!}$

   c. $\sum_{n=1}^{\infty} \frac{(-1)^n 3^{n-2}}{2^n}$

   d. $\sum_{n=1}^{\infty} \frac{\ln n}{n^2}$

2. Find all the values for x for which the series converges, and write as an interval. What is the radius of convergence?

   a. $\sum_{n=1}^{\infty} \frac{(-1)^n (x+1)^n}{n}$

   b. $\sum_{n=0}^{\infty} \frac{(x+1)^n}{n!}$

   c. $\sum_{n=0}^{\infty} 2\left(\frac{x}{8}\right)^{3n}$

3. Find the interval of convergence of the power series. Be sure to check for convergence at the endpoints of the intervals.

   a. $\sum_{n=0}^{\infty} \left(\frac{x}{2}\right)^n$

   b. $\sum_{n=0}^{\infty} \frac{(-1)^n n!(x-4)^n}{3^n}$

   c. $\sum_{n=1}^{\infty} \frac{n(-2x)^{n-1}}{n+1}$

   d. $\sum_{n=1}^{\infty} \frac{(-3)^n x^n}{n\sqrt{n}}$

   e. $\sum_{n=1}^{\infty} \frac{x^n}{n3^n}$

   f. $\sum_{n=1}^{\infty} n!\,(2x-1)^n$

   g. $\sum_{n=2}^{\infty} \frac{x^{2n}}{n(\ln n)^2}$

   h. $\sum_{n=1}^{\infty} \frac{(-1)^n x^n}{(n+1)(n+2)}$

   i. $\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{n!}$

   j. $\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{n!(n+1)!2^{2n+1}}$

   k. $\sum_{n=1}^{\infty} \frac{3^n (x+4)^n}{\sqrt{n}}$

   l. $\sum_{n=1}^{\infty} \frac{(2x-1)^n}{5^n \sqrt{n}}$

   m. $\sum_{n=2}^{\infty} \frac{5^n}{\ln n}(x-3)^n$

4. Find a geometric power series for the functions, centered at $c = 0$ unless a different c is specified.

   a. $f(x) = \frac{1}{2-x}$

   b. $f(x) = \frac{3}{2x-1}, c = 2$

   f. $f(x) = \frac{3}{2x-1}$

   g. $f(x) = \frac{3x}{x^2+x-2}$

c. $f(x)=\dfrac{1}{1-x^2}$

d. $f(x)=\dfrac{x}{(1-x)^2}$

e. $f(x) = \arctan x$

h. $f(x)=\ln(x+1)$

i. $f(x)=\dfrac{7x^3}{(1+3x^2)^4}$

5. Use a power series to approximate the definite integral to 6 decimal places.
   a. $\int_0^{0.2} \frac{1}{1+x^5}\,dx$
   b. $\int_0^{0.1} x \arctan 3x\,dx$

6. It can be useful when determining the convergence of a series to know which functions blow up more quickly (or go to zero more quickly), which tend to dominate in a relationship as $n \to \infty$. Some common functions have been shown to have the following dominance relationships (note: a, b are constants, and both greater than one):
$$\ln^a n \ll n^a \ll n^a \ln^b n \ll a^n \ll n! \ll n^n$$
What this means is that if we have an expression like $\frac{n!}{n^n}$ or $\frac{n^n}{n!}$ will be controlled by the fate of n^n once n gets very large. We can establish this relationship by using the ratio test (or the root test). What we'd like to do is establish the dominance relationships of 6 additional functions: $(2n)!, (n!)^2, n^{2n}, a^{n^2}, (n^2)!, n^{n^2}$. Construct ratios of these functions with each other and with functions like n^n in order to determine their dominance relations to extend the relationship listed above.

7. Use Taylor series to find the value of $\frac{1}{\sqrt{2\pi}}\int_{-1}^{1} e^{-\frac{x^2}{2}}\,dx$, i.e, $P(-1 \le X \le 1)$ for the standard normal distribution, to 4 decimal places.