MTH 264, Homework #6, Summer 2022 Name ____

Instructions: Write your work up neatly and attach to this page. Record your final answers (only) directly on this page if they are short; if too long indicate which page of the work the answer is on and mark it clearly. Use exact values unless specifically asked to round.

1. Determine the convergence or divergence of the series. State which test you used.
   a. $\sum_{n=1}^{\infty} \frac{\ln n}{n^2}$
   b. $\sum_{n=1}^{\infty} \frac{n}{n^4+1}$
   c. $\sum_{n=1}^{\infty} \left(\frac{\sin n}{n}\right)^2$
   d. $\sum_{n=1}^{\infty} \frac{2}{\sqrt{n^{\pi}}}$
   e. $\sum_{n=1}^{\infty} \left(\frac{1}{n^2} - \frac{1}{n^3}\right)$
   f. $\sum_{n=2}^{\infty} \frac{1}{n\sqrt{\ln n}}$
   g. $\sum_{n=3}^{\infty} \frac{1}{n \ln n \ln(\ln n)}$
   h. $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$
   i. $\sum_{n=1}^{\infty} \frac{1}{n\sqrt[3]{(\ln n)^2}}$

2. Find the values for p so that the series converges.
   a. $\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^p}$
   b. $\sum_{n=1}^{\infty} \frac{n}{(1+n^2)^p}$
   c. $\sum_{n=2}^{\infty} \frac{\ln n}{n^p}$

3. Find N such that $R_N \le 0.001$ for the convergent series.
   a. $\sum_{n=1}^{\infty} \frac{1}{n^4}$
   b. $\sum_{n=1}^{\infty} \frac{2}{n^2+5}$
   c. $\sum_{n=2}^{\infty} e^{-n/2}$

4. Use the fact that $\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$ and $\sum_{n=1}^{\infty} \frac{1}{n^4} = \frac{\pi^4}{90}$ to find the sum of the following series.
   a. $\sum_{n=2}^{\infty} \frac{1}{n^2}$
   b. $\sum_{n=1}^{\infty} \left(\frac{3}{n}\right)^4$
   c. $\sum_{n=3}^{\infty} \frac{1}{(n+1)^2}$
   d. $\sum_{n=5}^{\infty} \frac{1}{(n-2)^4}$
   e. $\sum_{n=1}^{\infty} \frac{1}{(2n)^2}$

5. Use the direct comparison test to determine the convergence of the series.
   a. $\sum_{n=1}^{\infty} \frac{1}{3n^2+2}$
   c. $\sum_{n=1}^{\infty} \frac{3^n}{4^n+5}$

b. $\sum_{n=1}^{\infty} \frac{1}{4\sqrt[3]{n}-1}$

6. Use the limit comparison test to determine the convergence of the series.

   a. $\sum_{n=1}^{\infty} \frac{2}{3^n - 5}$

   b. $\sum_{n=1}^{\infty} \tan\left(\frac{1}{n}\right)$

   c. $\sum_{n=1}^{\infty} \frac{1}{n(n^2+1)}$

7. For each of the series below, use at least two of the tests to determine the convergence of the series. Each test should be used at least once. Choose from: nth-term test, p-series test, integral test, limit comparison test, geometric series test, telescoping series test, direct comparison test. Clearly indicate which tests you are using.

   a. $\sum_{n=1}^{\infty} \frac{\sqrt{n}}{n}$

   b. $\sum_{n=1}^{\infty} \frac{1}{3^n + 2}$

   c. $\sum_{n=1}^{\infty} \frac{n}{2n+3}$

   d. $\sum_{n=1}^{\infty} \frac{n}{\left(n^2+1\right)^2}$

   e. $\sum_{n=1}^{\infty} \frac{n-1}{n^2\sqrt{n}}$

   f. $\sum_{n=1}^{\infty} \frac{n+4^n}{n+6^n}$

   g. $\sum_{n=1}^{\infty} \frac{1}{n^{1+\frac{1}{n}}}$

   h. $\sum_{n=0}^{\infty} 5\left(-\frac{1}{5}\right)^n$

   i. $\sum_{n=4}^{\infty} \frac{1}{3n^2 - 2n - 15}$

   j. $\sum_{n=1}^{\infty} \left(\frac{1}{n+1} - \frac{1}{n+2}\right)$

   k. $\sum_{n=1}^{\infty} \frac{3}{n(n+3)}$

   l. $\sum_{k=1}^{\infty} \frac{(2k-1)(k^2-1)}{(k+1)(k^2+4)}$

   m. $\sum_{n=1}^{\infty} \frac{e^{1/n}}{n}$

   n. $\sum_{n=1}^{\infty} \sin\frac{1}{n}$

8. Determine the convergence or divergence of the series. If the series converges, does it converge absolutely or conditionally?

   a. $\sum_{n=1}^{\infty} \frac{(-1)^n n^2}{n^2+1}$

   b. $\sum_{n=1}^{\infty} \cos n\pi$

   c. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}\sqrt{n}}{\sqrt[3]{n}}$

   d. $\sum_{n=0}^{\infty} (-1)^n e^{-n^2}$

   e. $\sum_{n=1}^{\infty} (-1)^{n+1} \arctan n$

   f. $\sum_{n=1}^{\infty} (-1)^n \sin\left(\frac{\pi}{n}\right)$

   i. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1} \ln(n+1)}{n+1}$

   j. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}\sqrt{n}}{n+2}$

   k. $\sum_{n=2}^{\infty} \frac{(-1)^n}{\ln n}$

   l. $\sum_{n=0}^{\infty} \frac{\cos n\pi}{n+1}$

   m. $\sum_{n=1}^{\infty} (-1)^{n-1} e^{2/n}$

   n. $\sum_{n=1}^{\infty} \frac{(-1)^n n^n}{n!}$

g. $\sum_{n=1}^{\infty}(-1)^n\left(\sqrt{n+1}-\sqrt{n}\right)$

h. $\sum_{n=0}^{\infty}\frac{(-3)^n}{(2n+1)!}$

o. $\sum_{n=1}^{\infty}(-1)^{n-1}ne^{-n}$

9. Determine the number of terms needed to approximate the sum of the convergent series with an error of less than 0.001.

a. $\sum_{n=0}^{\infty}\frac{(-1)^n}{2^n n!}=\frac{1}{\sqrt{e}}$

b. $\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n^2}$

c. $\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n4^n}=\ln\left(\frac{5}{4}\right)$

10. Use the root or ratio tests to determine the convergence or divergence of the series. In the test is inconclusive, use another test. State which test(s) you use.

a. $\sum_{n=1}^{\infty}\left(\frac{3}{4}\right)^n\left(\frac{1}{n!}\right)$

b. $\sum_{n=0}^{\infty}\frac{3^n}{(n+1)^n}$

c. $\sum_{n=1}^{\infty}\frac{1}{\sqrt{n}}$

d. $\sum_{n=1}^{\infty}\frac{n}{4^n}$

e. $\sum_{k=1}^{\infty}k\left(\frac{2}{3}\right)^k$

f. $\sum_{n=1}^{\infty}\frac{\cos\left(\frac{n\pi}{3}\right)}{n!}$

g. $\sum_{n=1}^{\infty}\frac{(2n)!}{(n!)^2}$

h. $\sum_{n=1}^{\infty}\frac{2^n n!}{5\cdot 8\cdot 11\cdots(3n+2)}$

i. $\sum_{n=1}^{\infty}\frac{n^2+1}{n!}$

j. $\sum_{n=0}^{\infty}\frac{(-1)^n 2^{4n}}{(2n+1)!}$

k. $\sum_{n=2}^{\infty}\frac{(-1)^n}{(\ln n)^n}$

l. $\sum_{n=1}^{\infty}\frac{(n!)^n}{(n^n)^2}$

m. $\sum_{n=1}^{\infty}\frac{\sin 4n}{4^n}$

n. $\sum_{n=1}^{\infty}\left(1+\frac{1}{n}\right)^{n^2}$

o. $\sum_{n=1}^{\infty}\frac{2^{n^2}}{n!}$