MTH 264, Homework #5, Summer 2022 Name _____

Instructions: Write your work up neatly and attach to this page. Record your final answers (only) directly on this page if they are short; if too long indicate which page of the work the answer is on and mark it clearly. Use exact values unless specifically asked to round.

1. Determine whether the sequence with the given nth term is monotonic. If it is not monotonic for the whole sequence, is it monotonic after a particular finite value of n? Is the sequence bounded above? Below? Use a graphing utility to confirm and sketch the graph. Does the sequence converge, and if so, to what value?

   a. $a_n = \dfrac{3n}{n+2}$

   b. $c_n = \left(-\dfrac{2}{3}\right)^n$

   c. $e_n = \dfrac{\sin n}{n}$

   d. $a_n = \sqrt{\frac{n+1}{9n+1}}$

   e. $a_n = \arctan(\ln n)$

   f. $a_n = n - \sqrt{n+1}\sqrt{n+3}$

   g. $b_n = ne^{-n/2}$

   h. $d_n = \cos\left(\dfrac{n\pi}{2}\right)$

   i. $a_n = \tan\left(\frac{2n\pi}{1+8n}\right)$

   j. $a_n = \frac{(-1)^{n+1}n}{n+\sqrt{n}}$

   k. $a_n = \sqrt[n]{2^{1+3n}}$

2. Determine the convergence or divergence of the series. If it is a telescoping or geometric series, find the limit of the sum. State which test you used.

   a. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n(n+2)}$

   b. $\displaystyle\sum_{n=1}^{\infty} \left(\sin n^n\right)$

   c. $\displaystyle\sum_{n=1}^{\infty} \ln\left(\frac{n+1}{n}\right)$

   d. $\sum_{n=1}^{\infty} \frac{1}{n^2+n^3}$

   e. $\sum_{n=1}^{\infty} \left(\cos\frac{1}{n^2} - \cos\frac{1}{(n+1)^2}\right)$

   f. $\sum_{n=2}^{\infty} \frac{1}{n^3-n}$

   g. $\displaystyle\sum_{n=0}^{\infty} 5\left(-\frac{1}{3}\right)^{n-1}$

   h. $\displaystyle\sum_{n=1}^{\infty} \frac{n+1}{2n-1}$

   i. $\displaystyle\sum_{n=0}^{\infty} \frac{4}{2^n}$

   j. $\sum_{n=2}^{\infty} \ln\left(\frac{n}{n+1}\right)$

   k. $\sum_{n=2}^{\infty} \frac{2}{n^2-1}$

   l. $\sum_{n=1}^{\infty} \left(e^{\frac{1}{n}} - e^{\frac{1}{n+1}}\right)$

3. Write the number as a ratio of integers.

   a. $2.\overline{516}$ b. $7.\overline{12345}$ c. $10.1\overline{35}$

4. List the first five terms of the sequence

   a. $a_n = \frac{1}{(n+1)!}$

   b. $a_n = \cos\left(\frac{n\pi}{2}\right)$

   c. $a_n = \frac{(-1)^n n}{n!+1}$

   d. $a_1 = 2, a_2 = 1, a_{n+1} = a_n - a_{n-1}$

5. Find a formula for the general term a_n of the sequence.

a. $\left\{1, -\frac{1}{3}, \frac{1}{9}, -\frac{1}{27}, \frac{1}{81}, \ldots\right\}$

b. $\left\{\frac{1}{2}, -\frac{4}{3}, \frac{9}{4}, -\frac{16}{5}, \frac{25}{6}, \ldots\right\}$

c. $\{0,1,3,6,10,15,21,\ldots.\}$

d. $\{5, 8, 11, 14, 17, \ldots\}$

e. $\{3^2, 3^3, 3^5, 3^9, 3^{17}, \ldots\}$

f. $\{1, 6, 120, 5040, 362880, \ldots.\}$

6. Evaluate the following expressions.

a. $7!$

b. $0!$

c. $\binom{12}{3}$

d. $\frac{10!}{3!}$

e. $\binom{4}{2}$

f. $\frac{10!}{4!6!}$

g. $\binom{10}{5}$

7. Calculate the first eight partial sums of the series. Does it appear to converge?

a. $\sum_{n=1}^{\infty} \frac{1}{n^3}$

b. $\sum_{n=1}^{\infty} \frac{n}{\sqrt{n^2+4}}$

c. $\sum_{n=2}^{\infty} \frac{1}{n(n+2)}$

d. $\sum_{n=1}^{\infty} \frac{1}{\ln n+1}$

e. $\sum_{n=1}^{\infty} \left(\frac{1}{\sqrt{n}} - \frac{1}{\sqrt{n+1}}\right)$

f. $\sum_{n=1}^{\infty} \sqrt[n]{2}$

8. Determine if the geometric series is convergent. If it is, find the sum. If it is not, explain why not.

a. $\sum_{n=1}^{\infty} \frac{(-3)^{n-1}}{4^n}$

b. $\sum_{n=0}^{\infty} \frac{e^n}{3^{n-1}}$

c. $3 - 4 + \frac{16}{3} - \frac{64}{9} + \cdots$

d. $\sum_{n=0}^{\infty} \frac{1}{(\sqrt{2})^n}$

e. $\sum_{k=1}^{\infty} (\cos 1)^k$

f. $\frac{1}{3} + \frac{2}{9} + \frac{1}{27} + \frac{2}{81} + \frac{1}{243} + \frac{2}{729} + \cdots$

9. Find the values of x for which the series converges.

a. $\sum_{n=0}^{\infty} 4\left(\frac{x-3}{4}\right)^n$

b. $\sum_{n=0}^{\infty} \frac{2^n}{x^n}$

c. $\sum_{n=0}^{\infty} e^{nx}$

d. $\sum_{n=1}^{\infty} (-5)^n x^n$

e. $\sum_{n=0}^{\infty} \frac{\sin^n x}{3^n}$