MTH 264, Homework #3, Summer 2022 Name ____

Instructions: Write your work up neatly and attach to this page. Record your final answers (only) directly on this page if they are short; if too long indicate which page of the work the answer is on and mark it clearly. Use exact values unless specifically asked to round.

1. Use integration by parts to evaluate the integral.
   a. $\int x\cos 5x\ dx$
   b. $\int (\ln x)^2\, dx$
   c. $\int t\cosh t\, dt$
   d. $\int x^3 e^{-x^2} dx$
   e. $\int s2^s\, ds$
   f. $\int \arctan 4t\ dt$
   g. $\int (x^2+1)e^{-x}dx$
   h. $\int_0^1 \frac{r^3}{\sqrt{4+r^2}}dr$
   i. $\int_0^\pi e^{\cos t}\sin 2t\, dt$

2. Verify the formula $\int x^n \ln x dx = \frac{x^{n+1}}{(n+1)^2}\left[-1+(n+1)\ln x\right]+C$.

3. Use the tabular method to evaluate each integral.
   a. $\int z^3 e^z dz$
   b. $\int t^4 \sinh t\, dt$
   c. $\int p^5 \cos p\, dp$

4. Use integration by parts to evaluate the integrals. Note: these integrals "loop".
   a. $\int e^{2\theta}\sin 3\theta d\theta$
   b. $\int \sec^3\theta\, d\theta$

5. Integrate using trigonometric identities.
   a. $\int \cos\theta\cos^5(\sin\theta)\, d\theta$
   b. $\int \frac{dx}{\cos x - 1}$
   c. $\int \frac{\cos x+\sin 2x}{\sin x}dx$
   d. $\int_0^{\frac{\pi}{4}} \sqrt{1-\cos 4\theta}\, d\theta$
   e. $\int \frac{\sin^3\sqrt{x}}{\sqrt{x}}dx$
   f. $\int \sec^4 q\, dq$
   g. $\int \frac{1}{\tan x+1}dx$

6. Apply identities to each integral to prepare it for integration and state the substitution to be used to integrate. Then use that substitution to rewrite the integral. You do not need to integrate.
   a. $\int \tan^5\phi\sec^4\phi\, d\phi$
   b. $\int \cot^5\phi\csc^9\phi\, d\phi$
   c. $\int \sin^4\alpha\cos^8 2\alpha\, d\alpha$
   d. $\int \cos^{10}\beta\, d\beta$
   e. $\int \cos^{17}\theta\sin^6\theta\, d\theta$
   f. $\int \sin^6\psi\cos^{12}\psi\, d\psi$
   g. $\int \cos^{11}\alpha\sin^3 4\alpha\, d\alpha$

7. Integrate.
   a. $\int \frac{x(x-2)}{(x-1)^3}dx$
   b. $\int \frac{1}{x\ln(x^3)}dx$
   g. $\int \frac{\sec x\tan x}{\sec x - 1}dx$
   h. $\int_{-2}^{2} \frac{dx}{x^2+4x+13}$

c. $\int \frac{1}{(x-1)\sqrt{x^2-2x}}dx$

d. $\int \frac{\sinh(x)}{1+\sinh^2(x)}dx$

e. $\int \tan(x)\ln(\cos x)dx$

f. $\int \frac{1}{3t+1} - \frac{17}{(4t-1)^2}dt$

i. $\int \frac{\arccos(x)}{\sqrt{1-x^2}}dx$

j. $\int \frac{5}{3e^x-2}dx$

k. $\int \frac{1}{(x-1)\sqrt{4x^2-8x+3}}dx$

l. $\int 3^t dt$

8. Use trig substitution to integrate the following. Be sure you convert back to the original variable.

a. $\int x^3\sqrt{1-x^2}dx$

b. $\int \frac{\sqrt{1+x^2}}{x}dx$

c. $\int \frac{du}{u\sqrt{5-u^2}}$

d. $\int x\sqrt{1-x^4}dx$

e. $\int_0^{\pi/2} \frac{\cos t}{\sqrt{1+\sin^2 t}}dt$

f. $\int \frac{dx}{\sqrt{x^2+16}}$

g. $\int \frac{x}{\sqrt{x^2-7}}dx$

h. $\int \frac{x}{\sqrt{x^2+x+1}}dx$

i. $\int \frac{x^2}{(3+4x-4x^2)^{\frac{3}{2}}}dx$

j. $\int x^2(x^2+1)^{3/2}dx$

9. Use partial fractions to rewrite each integral and then evaluate it. Be wary of situations where it may be necessary to do long division first.

a. $\int \frac{1+6x}{(4x-3)(2x+5)}$

b. $\int \frac{1}{(x^2-9)^2}dx$

c. $\int \frac{t^6+1}{t^6+t^3}dt$

d. $\int \frac{x^5+x-1}{x^3+1}dx$

e. $\int \frac{e^{2x}}{e^{2x}+3e^x+2}dx$

f. $\int \frac{x^4+1}{x^5+4x^3}dx$

g. $\int \frac{x^4-2x^3+x^2+2x-1}{x^2-2x+1}dx$

h. $\int \frac{4x}{x^3+x^2+x+1}dx$

i. $\int \frac{x^3+2x}{x^4+4x^2+3}dx$

10. Rewrite the following expressions using partial fractions. You do not need to find the constants, but do evaluate each term of the integral in terms of the unknown constants.

a. $\int \frac{dx}{(x+3)^2(x-2)(x^2+4)x^3}$

b. $\int \frac{dx}{(x^2-4)(x^2+7)^2(x-1)^3(x+1)}$

c. $\int \frac{dx}{(x^2+2x+2)(x^2+4x+3)^2(x-1)(x+2)^5}$

11. Make a substitution to convert each expression to a rational function. You don't need to integrate.

a. $\int \frac{\sqrt{x+1}}{x}dx$

b. $\int \frac{x^3}{\sqrt[3]{x^2+1}}dx$

e. $\int \frac{1}{1+\sqrt[3]{x}}dx$

f. $\int \frac{\sqrt{x}}{x^2+x}dx$

c. $\int \frac{1}{\sqrt{x}+\sqrt[3]{x}}\,dx$, use $u = \sqrt[6]{x}$.

d. $\int \frac{dx}{1+e^x}$

g. $\int \frac{\sqrt{1+\sqrt{x}}}{x}\,dx$