

**Instructions:** Write your work up neatly and attach to this page. Record your final answers (only) directly on this page if they are short; if too long indicate which page of the work the answer is on and mark it clearly. Use exact values unless specifically asked to round.

1. Find the average value of the function on the indicated interval.
  - a.  $f(x) = \frac{x^2 + 4}{x}, [1, 4]$
  - b.  $f(x) = \frac{2x}{(1+x^2)^2}, [0, 2]$
  - c.  $f(t) = e^{\sin t} \cos t, \left[0, \frac{\pi}{2}\right]$
  
2. Consider a demolition crane with a 500-pound ball suspended from a 40-foot cable that weighs 1 pound per foot. Find the work required to wind up all 40 feet of the apparatus.
  
3. Find the centroid of the lamina of uniform density bounded by the graphs, with density  $\rho = k$ .
  - a.  $x = 2y - y^2, x = 0$
  - b.  $x + y = 2, x = y^2$
  - c.  $y = e^x, y = 0, x = 0, x = 1$
  
4. A rectangular tank with a base 4 feet by 5 feet and a height of 4 feet is full of water. The water weighs 62.4 pounds per cubic foot. How much work is done in pumping water out over the top edge in order to empty
  - a. half of the tank?
  - b. all of the tank?
  
5. A mining company estimates that the marginal cost of extracting  $x$  tons of copper from a mine is  $C'(x) = 0.6 + 0.008x$  measured in thousands of dollars. If start-up costs are \$100,000, what is the cost of extracting the first 50 tons of copper? What about the next 50?
  
6. A force of 10 lbs. is required to hold a spring 4 in. beyond its natural length. How much work is done in stretching the spring from its natural length to 6 in beyond its natural length?
  
7. A chain lying on the ground is 10m long and its mass is 80 kg. How much work is required to raise one end of the chain to a height of 6m?
  
8. An aquarium 2m long, and 1m wide, and 1m deep is full of water. Find the work needed to pump half the water out of the aquarium. Water density is  $1000 \frac{kg}{m^3}$ .
  
9. A circular swimming pool had a diameter of 24 ft., the sides of the pool are 5 ft. high and the water is 4 ft. deep. How much work is required to pump all of the water out over the side? Water density is  $62.5 \frac{lbs}{ft^3}$ .
  
10. Find the average value of the function  $f(x) = (x - 3)^3$  on the interval  $[2, 5]$ . Then find the value of  $c$  for which  $f(c) = \bar{f}$ .

11. If the revenue flows into a company at a rate of  $f(t) = 9000\sqrt{1 + 2t}$  where  $t$  is measured in years and  $f(t)$  in dollars. Find the total revenue in the first 4 years.

12. Let  $f(x) = \frac{c}{1+x^2}$ . For what values of  $c$  is  $f(x)$  a probability density function?

13. If  $f(x) = 30x^2(1 - x)^2, 0 \leq x \leq 1$  is a probability density function. Find:

a.  $P(0 \leq X \leq 1)$

d.  $P\left(X \leq \frac{1}{3}\right)$

b. The mean, i.e. calculate  $\int_0^1 xf(x)dx = \mu$

c. Use that information to find the variance  $\sigma^2 = \int_0^1 (x - \mu)^2 f(x)dx$