MTH 264, Homework #1, Summer 2022 Name ____

Instructions: Write your work up neatly and attach to this page. Record your final answers (only) directly on this page if they are short; if too long indicate which page of the work the answer is on and mark it clearly. Use exact values unless specifically asked to round.

1. Find the area bounded by the curves and sketch the region.
   a. $f(x) = x^4 - 4x^2$ and $g(x) = x^3 - 4x$
   b. $y = x^2 - 2x, y = x + 4$
   c. $y = \cos x\,, y = 2 - \cos x\,, [0, 2\pi]$
   d. $y = x^3, y = x$
   e. $y = |x|, y = x^2 - 2$
   f. $y = e^x, y = x^2 - 1, x = -1, x = 1$
   g. $x = 1 - y^2, x = y^2 - 1$
   h. $x = y^4, y = \sqrt{2x}, y = 0$
   i. $y = \frac{1}{x}, y = x, y = \frac{1}{4}x, x > 0$
   j. $y = x^2e^{-x}, y = xe^{-x}$

2. Set up the integral to find the value of the area bounded by the graphs. You do not need to evaluate it, but do sketch the region.
   a. $y = \frac{2}{1+x^4}, y = x^2$
   b. $y = \cos x\,, y = x + 2\sin^4 x$
   c. $y = e^{1-x^2}, y = x^4$

3. Calculate the volumes of the solids of revolution as indicated below. Calculate the volume both with the shell method and with the disk or washer method and verify that the volume is the same in each case.
   a. $y = 2x^2, y = 0, x = 2$ Revolved around the:
      i. y-axis
      ii. the line $y = 8$
      iii. x-axis
      iv. the line $x = 2$
   b. $y = \dfrac{10}{x^2}, y = 0, x = 1, x = 5$ Revolved around:
      i. y-axis
      ii. $y = 10$
      iii. x-axis
      iv. $x = 5$
   c. $y = e^{-x}, y = 1, x = 2$ Revolved around:
      i. y-axis
      ii. x-axis
      iii. $y = 0$
      iv. $x = 3$
   d. $x = y^2, x = 1 - y^2$ Revolved around:
      i. $y = 1$
      ii. $x = 3$
   e. $y = x^3, y = 0, x = 1$ Revolved around:
      i. y-axis
      ii. x-axis
      iii. $x = 1$
      iv. $y = 1$

4. Find the surface area when the function $y = \dfrac{x^3}{6} + \dfrac{1}{2x}$ on the interval $1 \le x \le 2$.

5. A torus is formed by revolving the region bounded by the circle $x^2 + y^2 = 4$ around the line x=3. Find the volume of this doughnut-shaped solid. [Hint: find the function that represents the semicircle.]

6. A tank on the wing of a jet aircraft is formed by revolving the region bounded by the graph of $y = \frac{1}{8}x^2\sqrt{2-x}$ and the *x*-axis, around the *x*-axis, where *x* and *y* are measured in meters. Find the tank's volume.

7. An ornamental light bulb is designed by resolving the graph of $y = \frac{1}{3}x^{1/2} - x^{3/2}$ on the interval [0,1/3] around the x-axis, where x and y are measured in feet. Find the surface area of the bulb and use the result to approximate the amount of glass needed to make the bulb (assume that the glass is 0.015 inches thick).

8. Find the volume of the region bounded by $y = e^x, y = e^{-x}, x = 1$ around the y-axis. (Use shells.)

9. Differentiate.

   a. $F(x) = \int_1^{x^2} \frac{1}{t} dt$

   b. $h(x) = \int_0^{x^4} t\cos(t^2)\,dt$

   c. $y(x) = e^{x^2} \int_0^x e^{-t^2} dt$

   d. $u(t) = e^{\cos(\sin^2 3t)}$

   e. $q(t) = \frac{\cos(\ln t)}{t}$

   f. $v(t) = \tan\left(\frac{1}{t}\right)$

   g. $g(x) = \int_x^1 \sqrt{t + \sin t}\,dt$

   h. $r(x) = \int_{\sqrt{x}}^{x} \frac{e^t}{t} dt$

   i. $a(t) = 2^{t-1}$

   j. $p(x) = \sqrt{x^2 + 3}$

   k. $s(t) = \arcsin(e^t + t)$

10. Find the length of arc in rectangular coordinates. You may need to calculate the value numerically.

    a. $y = 1 + 6x^{\frac{3}{2}}, [0,1]$

    b. $y = (1 - e^{-x}), [0,2]$

    c. $y = \frac{x^3}{3} + \frac{1}{4x}, [1,2]$

    d. $y = x\sin x\,, [0,2\pi]$

    e. $y = \ln(\sec x)\,, \left[0, \frac{\pi}{4}\right]$

    f. $y = \sqrt[3]{x}, [0,1]$

    g. $y = \sqrt{x - x^2} + \arcsin x\,, [0,1]$

    h. $y = e^{-x^2}, [-1,1]$