MTH 265, Quiz #6, Summer 2021 Name KEY

Instructions: Show all work. Use exact answers unless specifically asked to round. Be sure to complete all parts of each question.

1. Find the volume of the solid bounded by $f(x,y) = -x^2 - y^2, +9, z = 0$. Set the integral up in rectangular and polar coordinates. Integrate the polar version.

$$\int_{-3}^{3}\int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}} -x^2 - y^2 + 9 \, dy\, dx$$

$$\int_0^{2\pi}\int_0^3 (-r^2+9)r\,dr\,d\theta = \int_0^{2\pi}\int_0^3 -r^3 + 9r\,dr\,d\theta =$$

$dydx = r\,dr\,d\theta$
$x^2 + y^2 = r^2 \Rightarrow r = 3$
$f(r,\theta) = -r^2 + 9$

$$\int_0^{2\pi} -\frac{r^4}{4} + \frac{9}{2}r^2 \Big|_0^3 d\theta = \int_0^{2\pi} -\frac{81}{4} + \frac{81}{2}\,d\theta = \frac{81}{4}\theta\Big|_0^{2\pi} =$$

$$\frac{81}{4}\cdot 2\pi = \frac{81\pi}{2}$$

2. Find the volume of the solid bounded above by the sphere of radius 9 centered at the origin, and below by the cone $z = \sqrt{x^2 + y^2}$. [Hint: it will be easier to integrate in spherical coordinates.]

$\rho\cos\varphi = \rho\sin\varphi$
$\varphi = \pi/4$

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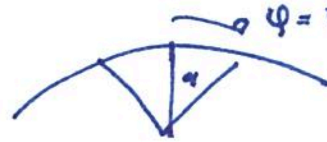

$$\int_0^{2\pi}\int_0^{\pi/4}\int_0^9 \rho^2 \sin\varphi \, d\rho\, d\varphi\, d\theta$$

$$\frac{\rho^3}{3}\Big|_0^9 = 243$$

$$\int_0^{2\pi}\int_0^{\pi/4} 243\sin\varphi\, d\varphi\, d\theta = \int_0^{2\pi} -243\cos\varphi \Big|_0^{\pi/4} d\theta = \int_0^{2\pi} -243(1/\sqrt{2} - 1)\,d\theta$$

$$= \left(243 - \frac{243}{\sqrt{2}}\right)2\pi$$

3. Find the potential function, if it exists, for the vector field $F(x,y,z) = (3x^2y - z)\hat{\imath} + (yz + x^3)\hat{\jmath} + (\frac{1}{2}y^2 - x)\hat{k}$. If it does not exist, verify this by applying the test for conservative vector fields. (on last quiz. it is conservative $\nabla \times F = \vec{0}$)

$$\int 3x^2y - z\,dx = x^3y - xz + G(y,z)$$
$$\int yz + x^3\,dy = \frac{1}{2}y^2z + x^3y + H(x,z)$$
$$\int \frac{1}{2}y^2 - \cancel{x}\,dz = \frac{1}{2}y^2z - xz + I(x,y)$$
$$f(x,y,z) = x^3y - xz + \frac{1}{2}y^2z + K$$