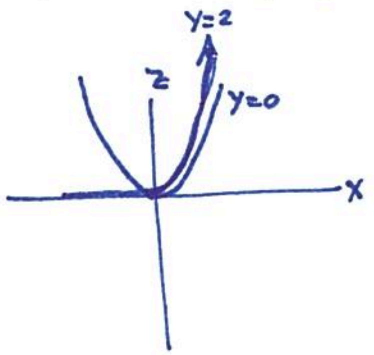
MTH 265, Quiz #4, Summer 2021 Name KEY

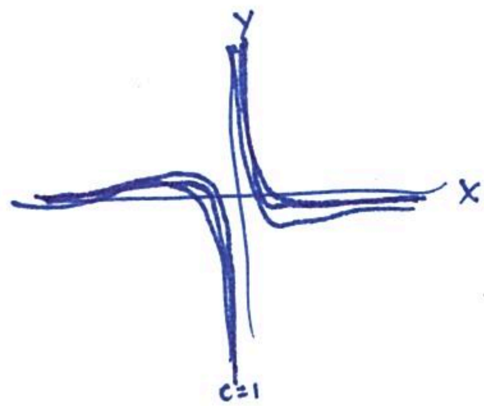
Instructions: Show all work. Use exact answers unless specifically asked to round. Be sure to complete all parts of each question.

1. For the function $f(x,y) = x^2 e^{xy/2}$, sketch the trace of the graph when y=0 and y=2. Sketch at least 5 level curves of the graph. Put the traces on one graph, and the five level curves on another.

$y=0 \qquad z = x^2$

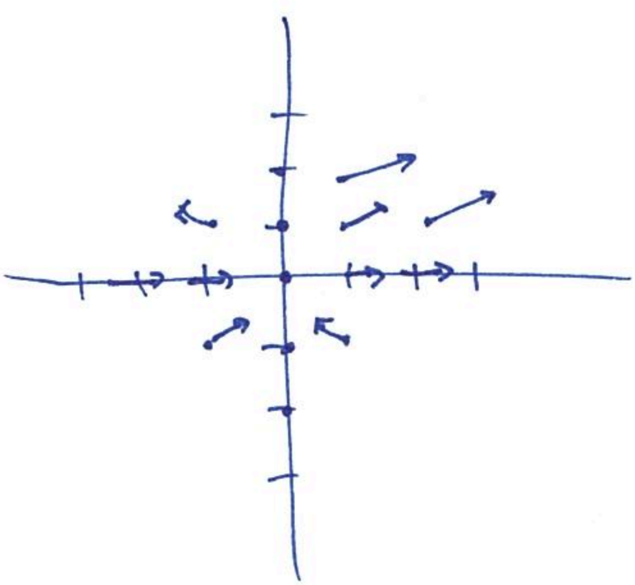
$y=2 \qquad z = x^2 e^x$





$\frac{c}{x^2} = e^{xy/2} \rightarrow \ln\left(\frac{c}{x^2}\right) = xy/2 \rightarrow \frac{2}{x}\ln\left(\frac{c}{x^2}\right) = y \qquad z \neq 0$

2. Sketch the vector field $\vec{F}(x,y) = 2xy\hat{\imath} + x^2\hat{\jmath}$. Plot at least 10 points or more to determine the general behavior of the field.



(0,0) <0,0>
(1,0) <0,1>
(-1,0) <0,1>
(0,1) <0,0>
(0,-1) <0,0>
(1,1) <2,1>
(-1,-1) <2,1>
(1,-1) <-2,1>
(-1,1) <-2,1>
(2,1) <4,2>
(1,2) <4,1>

3. Find the value of the line integral $\int_C (x + y^2)ds$ along the curve $\vec{r}(t) = \cos t\,\hat{\imath} + \sin t\,\hat{\jmath}$.

circle $0 \le t \le 2\pi$

$r'(t) = \langle -\sin t, \cos t \rangle$

$\|r'(t)\| = \sqrt{\sin^2 t + \cos^2 t} = 1$

$ds = \|r'(t)\|\,dt = dt$

$$\int_0^{2\pi} \cos t + \sin^2 t\, dt = \int_0^{2\pi} \cos t + \frac{1}{2} - \frac{1}{2}\sin 2t\, dt =$$

$$\cancel{\sin t} + \frac{1}{2}t + \cancel{\frac{1}{4}\cos 2t}\,\Big|_0^{2\pi} = \pi$$

4. Find the value of the line integral $\int_C (x + 2y)dx + (3x - y)dy$ along the curve $\vec{r}(t) = t\hat{\imath} + t^2\hat{\jmath}$.

$0 \le t \le 1$

$dx = dt$

$dy = 2t\,dt$

$$\int_0^1 (t + 2t^2)dt + (3t - t^2)2t\,dt$$

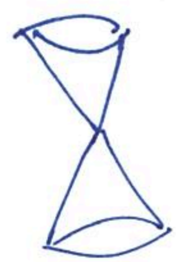
$$\int_0^1 t + 2t^2 + 6t^2 - 2t^3\,dt$$

$$\int_0^1 t + 8t^2 - 2t^3\,dt = \frac{1}{2}t^2 + \frac{8}{3}t^3 - \frac{2}{4}t^4\,\Big|_0^1$$

$$\cancel{\frac{1}{2}} + \frac{8}{3} - \cancel{\frac{1}{2}} = \frac{8}{3}$$

5. Determine what kind of surface is being modeled with the parametric function $\vec{r}(u, v) = u\cos v\,\hat{\imath} + u\sin v\,\hat{\jmath} + u\hat{k}$. Describe the surface in as much detail as possible or sketch the graph.

Cone around z-axis



$r = z$

$r^2 = z^2$

$x^2 + y^2 = z^2$

6. Write a vector-valued function for the surface described by $x = \sqrt{16y^2 + z^2}$.

$x = v$

$y = 4\sin u$

$z = \cos u$

$r(u,v) = \langle v, 4v\sin u, v\cos u \rangle$

or

$v = y$

$u = z$

$x = \sqrt{16v^2 + u^2}$

$r(u,v) = \langle \sqrt{16v^2 + u^2}, v, u \rangle$