

**Instructions:** Show all work. Use exact answers unless specifically asked to round. Be sure to complete all parts of each question.

1. Rewrite the equation  $x^2 + y^2 + z^2 - 9z = 0$  in cylindrical and spherical coordinates.

spherical  $\rho^2 - 9\rho \cos \phi = 0$

cylindrical  $r^2 + z^2 - 9z = 0$

2. Determine if the  $\lim_{(x,y) \rightarrow (0,0)} \frac{xy^2}{x^2+y^4}$  exists or is undefined. If it does exist, say what it is.

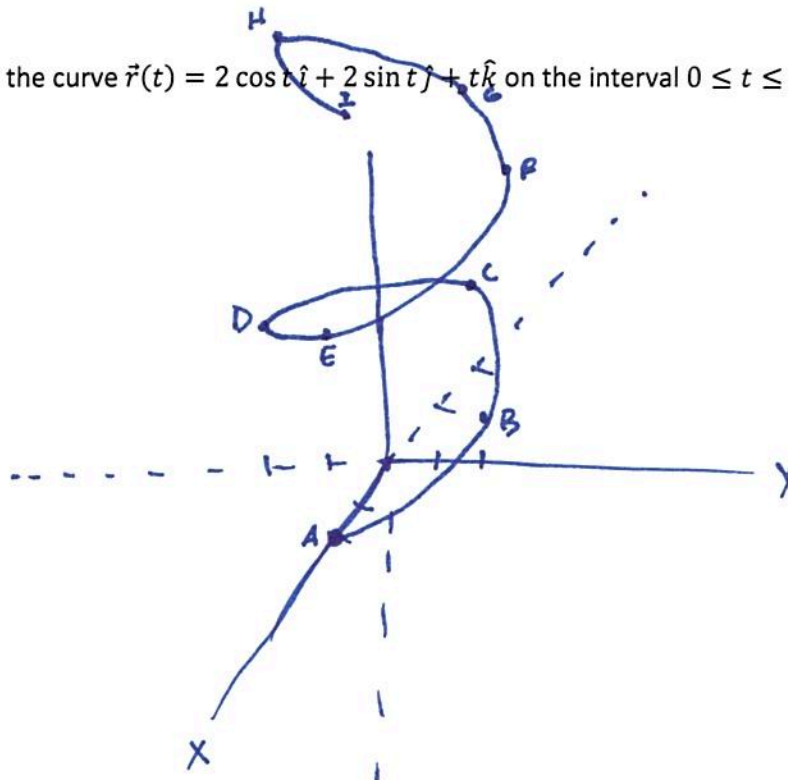
$$x^2 = y^4$$

$$x = ky^2$$

$$\lim_{y \rightarrow 0} \frac{ky^2 y^2}{k^2 y^4 + y^4} = \lim_{y \rightarrow 0} \frac{ky^4}{y^4(k^2+1)} = \frac{k}{k^2+1}$$

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3. Sketch the curve  $\vec{r}(t) = 2 \cos t \hat{i} + 2 \sin t \hat{j} + t \hat{k}$  on the interval  $0 \leq t \leq 4\pi$ .



- A  $t=0$   $(2, 0, 0)$
- B  $t=\pi/2$   $(0, 2, \pi/2)$
- C  $t=\pi$   $(-2, 0, \pi)$
- D  $t=3\pi/2$   $(0, -2, 3\pi/2)$
- E  $t=2\pi$   $(2, 0, 2\pi)$
- F  $t=5\pi/2$   $(0, 2, 5\pi/2)$
- G  $t=3\pi$   $(-2, 0, 3\pi)$
- H  $t=7\pi/2$   $(0, -2, 7\pi/2)$
- I  $t=4\pi$   $(2, 0, 4\pi)$

4. For the vector-valued functions  $\vec{r}(t) = 2 \cos t \hat{i} + 2 \sin t \hat{j} + t \hat{k}$  and  $\vec{u}(t) = t^3 \hat{i} + e^t \hat{j} - \frac{1}{t} \hat{k}$ , perform the indicated operations.

a.  $\vec{r}'(t)$

$$\langle -2 \sin t, 2 \cos t, 1 \rangle$$

b.  $\int \vec{u}(t) dt$

$$\langle \frac{1}{4} t^4 + C_1, e^t + C_2, -\ln|t| + C_3 \rangle$$

c.  $\vec{r}(t) \cdot \vec{u}(t)$

$$2t^3 \cos t + 2e^t \sin t - 1$$

d.  $\|\vec{r}(t)\|$

$$\sqrt{(2 \cos t)^2 + (2 \sin t)^2 + t^2} = \sqrt{4 + t^2}$$