MTH 265, Exam #3, Summer 2021 Name ____

Academic Integrity Statement

I affirm that, I, ____________________________ (student name), do attest that I alone am completing the problems on this test without receiving unauthorized assistance. I understand that violations of academic integrity may result in sanctions, up to and including expulsion from the college.

____________________________________(Student Signature)

____________________________________(Student ID number)

Attach a copy of your photo ID to the online submission (there is a question drop box for it). The ID must be a photo ID. A Driver's license, School ID (NOVA or otherwise), or a work ID are acceptable as long as it contains your full name and photo.

Instructions: Show all work. Use exact answers unless specifically asked to round. You may check your answers in the calculator, but you must show work to get full credit. Incorrect answers with no work will receive no credit. Be sure to complete all the requested elements of each problem.

1. For the curve $\vec{r}(t) = sin^3 t\hat{\imath} - cos^3 t\hat{\jmath}$, find the unit tangent vector, the unit normal vector and the binormal vector. (14 points)

2. Find the directional derivative of the function $f(x, y) = x^2 e^{x+y}$ at the point (1,-1,1) in the direction $\vec{u} = 2\hat{\imath} - 3\hat{\jmath}$. (8 points)

3. Find the equation of the tangent plane for the function $g(x, y) = y \ln|x| - x^2$ at the point (1,6,-1). Then find the equation of the line that is normal to the surface at the same point. (12 points)

4. Find the equation of the tangent plane for the parametric surface $\vec{r}(u, v) = 2\cos u \sin v\,\hat{\imath} + 2\sin u \sin v\,\hat{\jmath} + 2\cos v\,\hat{k}$ at the point $(\sqrt{2}, 0, \sqrt{2})$. Then find the equation for the normal vector to the function at any point. (12 points)

5. Find the length of the parametric curve $\vec{r}(t) = t\hat{\imath} + \ln|cos\, t|\, \hat{\jmath}$. (8 points)

6. Find the curvature of the function $\vec{r}(t) = t\sin t\,\hat{\imath} - \cos t\,\hat{\jmath} + t^2\hat{k}$. Use the formula $\kappa = \frac{\|\vec{r}\prime \times \vec{r}\prime\prime\|}{\|\vec{r}\prime\|^3}$. Note any values of the parameter where the curvature is zero. (12 points)

7. Find the surface area of the surface $x^2 + y^2 - z^2 = 1$ above the xy-plane and below the line $z = 4$. (10 points)

8. Find the value of $\int_C \vec{F} \cdot d\vec{r}$ for the vector field $\vec{F}(x,y,z) = \frac{1}{3}x^2 z\hat{\imath} + \frac{1}{2}y\hat{\jmath} + \frac{1}{9}x^3\hat{k}$ from the point (2,-1,0) to the point (1,1,3). If the field is conservative, use the Fundamental Theorem of line integrals. If it is not, find the work done on the straight-line path between the two points. (12 points)

9. Find the value of the line integral $\int_C (2x - y)dx + (y + 5x)dy$ around the area bounded by the rectangle with vertices (0,0), (2,0), (2,5), (0,5), traversed from (0,0) counterclockwise. [Hint: Use Green's Theorem.] (10 points)

10. Compute the value of the surface integral $\int\int_S g(x,y,z)dS$ for $g = xy$ over the triangle $x + y + z = 1, x, y, z \geq 0$. (10 points)

11. Find the value of the surface integral $\int\int_S \|\vec{r_u} \times \vec{r_v}\| dS$ for $\vec{r}(u,v) = uv\hat{\imath} + (u+v)\hat{\jmath} + (u-v)\hat{k}$, for $0 \leq u \leq 3, 1 \leq v \leq 5$. (12 points)

12. Find the flux $\int\int_S \vec{F} \cdot \vec{N} dS$ by the Divergence Theorem for $\vec{F}(x,y,z) = x^2\hat{\imath} + y^2\hat{\jmath} + z^2\hat{k}$ on the unit sphere. (10 points)

13. Use Stokes' Theorem to compute $\int_C \vec{F} \cdot d\vec{r} = \int\int_S (\nabla \times \vec{F}) \cdot \vec{N} dS$ for $\vec{F}(x, y, z) = x^2\hat{\imath} + y^2\hat{\jmath} + z^2\hat{k}$ over the top half of the unit sphere. (13 points)