Constant coefficients. Solutions

Directions: Do one step, and then pass it along to the next student. You do not have to solve the entire problem. If you see a mistake, correct it. If you are not sure, discuss. I will check back.

Solve each of the problems below. Explain how the solution methods differ.

1. $y'' + 3y' - 54y = 0$

Let $y = e^{kx}, y' = ke^{k}x, y'' = k^2e^{kx}$. Then the characteristic equation is $k^2 + 3k - 54 = 0 \rightarrow (k+9)(k-6) = 0, or\ k = -9, 6$. Thus $\boldsymbol{y = c_1e^{-9x} + c_2e^{6x}}$.

$$W = \begin{vmatrix} e^{-9x} & e^{6x} \\ -9e^{-9x} & 6e^{6x} \end{vmatrix} = 6e^{-3x} + 9e^{-3x} = \mathbf{15}\boldsymbol{e^{-3x}}$$

2. $y'' + 18y' + 81y = 0$

The characteristic equation is $k^2 + 18k + 81 = 0 \rightarrow (k+9)^2 = 0, k = -9$, Thus $\boldsymbol{y = c_1e^{-9x} + c_2xe^{-9x}}$.

$$W = \begin{vmatrix} e^{-9x} & xe^{-9x} \\ -9e^{-9x} & e^{-9x} - 9xe^{-9x} \end{vmatrix} = e^{-18x} - 9xe^{-18x} + 9xe^{-18x} = \boldsymbol{e^{-18x}}$$

3. $y'' + 2y' + 10y = 0$

The characteristic equation is $k^2 + 2k + 10 = 0$. This does not factor, so we use the quadratic formula. $k = \frac{-2\pm\sqrt{2^2-4(1)(10)}}{2(1)} = \frac{-2\pm\sqrt{4-40}}{2} = \frac{-2\pm 6i}{2} = -1 \pm 3i$

Thus, $y = \boldsymbol{c_1e^{-x}\sin(3x) + c_2e^{-x}\cos(3x)}$.

$$W = \begin{vmatrix} e^{-x}\sin 3x & e^{-x}\cos 3x \\ -e^{-x}\sin 3x + 3e^{-x}\cos 3x & -e^{-x}\cos 3x - 3e^{-x}\sin 3x \end{vmatrix} =$$
$$e^{-2x}\begin{vmatrix} \sin 3x & \cos 3x \\ -\sin 3x + 3\cos 3x & -\cos 3x - 3\sin 3x \end{vmatrix} =$$
$$e^{-2x}[-\sin 3x\cos 3x - 3\sin^2 3x + \sin 3x\cos 3x - 3\cos^2 3x] =$$
$$e^{-2x}[-3][\sin^2 3x + \cos^2 3x] = \boldsymbol{-3e^{-2x}}$$

4. Find the Wronskian for each of the fundamental sets above.

Shown above. May differ by sign.