Cauchy-Euler.

Directions: Do one step, and then pass it along to the next student. You do not have to solve the entire problem. If you see a mistake, correct it. If you are not sure, discuss. I will check back.

Solve each of the problems below. Explain how the solution methods differ.

1. $x^2y'' - 4xy' + 6y = 0$

Let $y = x^n, y' = nx^{n-1}, y'' = n(n-1)x^{n-2}$. The auxiliary equation is $n(n-1) - 4n + 6 = 0 \to n^2 - 5n + 6 = 0, (n-3)(n-2) = 0, n = 2, 3$. Thus the solution is $\boldsymbol{y = c_1x^2 + c_2x^3}$.

$$W = \begin{vmatrix} x^2 & x^3 \\ 2x & 3x^2 \end{vmatrix} = 3x^4 - 2x^4 = \boldsymbol{x^4}$$

2. $x^2y'' + 9xy' + 16y = 0$

The auxiliary equation is $n(n-1) + 9n + 16 = 0 \to n^2 + 8n + 16 = 0, (n+4)^2 = 0, n = 4$. Thus, the solution is $\boldsymbol{y = c_1x^4 + c_2x^4 \ln x}$.

$$W = \begin{vmatrix} x^4 & x^4 \ln x \\ 4x^3 & 4x^3 \ln x + x^3 \end{vmatrix} = 4x^7 \ln x + x^7 - 4x^7 \ln x = \boldsymbol{x^7}$$

3. $x^2y'' + 3xy' + 5y = 0$

The auxiliary equation is $n(n-1) + 3n + 5 = 0 \to n^2 + 2n + 5 = 0$. We must use the quadratic equation to solve since this is not factorable. $n = \frac{-2\pm\sqrt{2^2-4(1)(5)}}{2(1)} = \frac{-2\pm\sqrt{4-20}}{2} = \frac{-2\pm 4i}{2} = -1 \pm 2i$. Thus, the solution is $\boldsymbol{y = c_1x^{-1}\sin(2\ln x) + c_2x^{-1}\cos(2\ln x)}$.

$$W = \begin{vmatrix} x^{-1}\sin(2\ln x) & x^{-1}\cos(2\ln x) \\ -x^{-2}\sin(2\ln x) + 2x^{-2}\cos(2\ln x) & -x^{-2}\cos(2\ln x) - 2x^{-2}\sin(2\ln x) \end{vmatrix} =$$

$$-x^{-3}\sin(2\ln x)\cos(2\ln x) - 2x^{-3}\sin^2(2\ln x) + x^{-3}\sin(2\ln x)\cos(2\ln x) - 2x^{-3}\cos^2(2\ln x) =$$

$$-2x^{-3}[\sin^2(2\ln x) + \cos^2(2\ln x)] = \boldsymbol{-2x^{-3}}$$

4. Find the Wronskian for each of the fundamental sets above.

Work for Wronskians is above. Could differ by sign.