MTH 291, Practice Exam #2, Fall 2018   Name ____

Instructions: Show all work. Answers without work required to obtain the solution will not receive full credit. Some questions may contain multiple parts: be sure to answer all of them. Give exact answers unless specifically asked to estimate.

1. Solve the second-order ODEs for the general solution.
   a. $2y'' - y' - y = 0$
   b. $y'' - 2y' + 2y = 0$
   c. $y'' - 18y' + 81y = 0$

2. The table below gives the solution to the second order constant coefficient homogeneous equation, and the forcing function $F(x)\ or\ F(t)$. Determine the Ansatz for the method of undetermined coefficients in each case.

	y_1	y_2	$F(x)\ or\ F(t)$	Ansatz
a.	e^{-2x}	e^{3x}	$2\sin 3x$	
b.	$e^{-x}\cos x$	$e^{-x}\sin x$	$e^{x}\sin x$	
c.	$e^{-x/2}\sin\left(\frac{\sqrt{3}}{2}x\right)$	$e^{-x/2}\cos\left(\frac{\sqrt{3}}{2}x\right)$	$e^{x}+7$	
d.	e^{-t}	1	$t+e^{-t}$	
e.	$\sin t$	$\cos t$	$\cos^2 t$	

3. What is the difference between the natural frequency of the system, and a quasi-frequency? How is each obtained?

4. What conditions are needed in a forced oscillation system to achieve beats?

5. Use the method of reduction of order to solve $(1-x^2)y'' - 2xy' + 2y = 0$, given $y_1(x) = x$.

6. Set up the differential equation to solve the spring-mass problem with a 12 lbs. weight that stretches a spring 6 in. and a dashpot that provides 3 lbs. of resistance for every ft/s of velocity. The weight is pulled from an additional one foot from equilibrium and then released from rest.
   a. Is the system undamped, underdamped, critically damped or overdamped?
   b. Solve for an equation for the position of the mass at any time t.
   c. State the period (or quasi-period), amplitude and phase shift.
   d. What is the behavior of the system as $t \to \infty$?

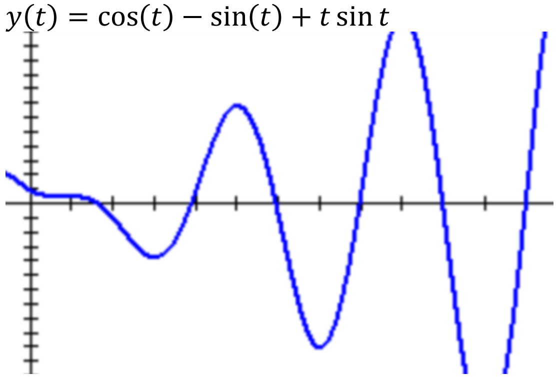
7. Use the method of variation of parameters to find the particular solution to $y'' + 6y' + 9y = 4e^{2t} + e^{-t}$.

8. Use the method of undetermined coefficients to find the particular solution to
   $y'' + 6y' + 9y = 4e^{2t} + e^{-t}$.

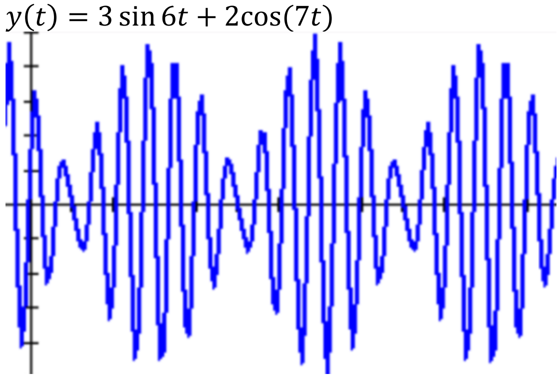
9. Use the method of undetermined coefficients to find the particular solution to
   $2y'' + 3y' + y = t^2 + 3\sin t\,, y(0) = 0, y'(0) = 1$.

10. Use the method of variation of parameters to find the particular solution to
    $y'' - 2y' + y = \frac{e^t}{1+t^2}$.

11. Below are the graphs of solutions to forced spring problems. Determine if the solution models resonance or beats (or neither). Explain your reasoning.
    a. $y(t) = \cos(t) - \sin(t) + t\sin t$



    b. $y(t) = 3\sin 6t + 2\cos(7t)$



12. Sketch a graph of what an overdamped spring system looks like.

13. For each of the solutions below to a forced oscillation system, state i) the transient or steady state solution, ii) whether the system is undamped, underdamped, critically damped or overdamped, and iii) if resonance or beats occurs.
    a. $y(t) = e^{-t}(c_1\cos 5t + c_2\sin 5t) + 5\cos 4t + 4\sin 4t$

    b. $y(t) = c_1e^{-t} + c_2e^{-2t} + \sin 3t$

    c. $y(t) = c_1\cos 2t + c_2\sin 2t + \frac{1}{6}t\cos 2t$

14. Use Abel's Theorem to find the value of the Wronskian for $y'' + 2xy' + 8y = 0$.

15. Find the Wronksian for $\{t^2, t^2 \ln t\}$.

16. Use the definition of the Laplace transform $\mathcal{L}\{f(t)\} = \int_0^\infty e^{-st} f(t)dt$ to find $\mathcal{L}\{1 + \cosh 5t\}$.

17. Use the table of Laplace transforms to find Laplace transforms or inverse Laplace transforms as indicated.
    a. $\mathcal{L}\{(1+t)^2\}$

    b. $\mathcal{L}\{te^t\}$

    c. $\mathcal{L}\{e^{-2t} \sin 3\pi t\}$

    d. $\mathcal{L}^{-1}\left\{\frac{1}{2} - \frac{2}{s^5}\right\}$

    e. $\mathcal{L}^{-1}\left\{\frac{9-17s}{s^2+81}\right\}$

    f. $\mathcal{L}^{-1}\left\{\frac{1}{s(s^2+4)}\right\}$

    g. $\mathcal{L}^{-1}\left\{\frac{1}{s^2(s^2-1)}\right\}$

18. Use Laplace transforms to solve the IVP $y'' + 4y' + 8y = e^{-t}, y(0) = 0,\ y'(0) = 1$.

19. Use the definition of the Laplace transform $\mathcal{L}\{f(t)\} = \int_0^\infty e^{-st} f(t)dt$ to find a formula for $\mathcal{L}\{f(t)\}$.

20. A mass of 100 g stretches a spring 5 cm. If the mass is set in motion from its equilibrium position with a downward velocity of 10 cm/sec, and there is no damping.
    a. Determine the position y of the mass at any time *t*.
    b. When does the mass first return to equilibrium? (i.e. when is y=0?)
    c. State the period, amplitude and phase shift.