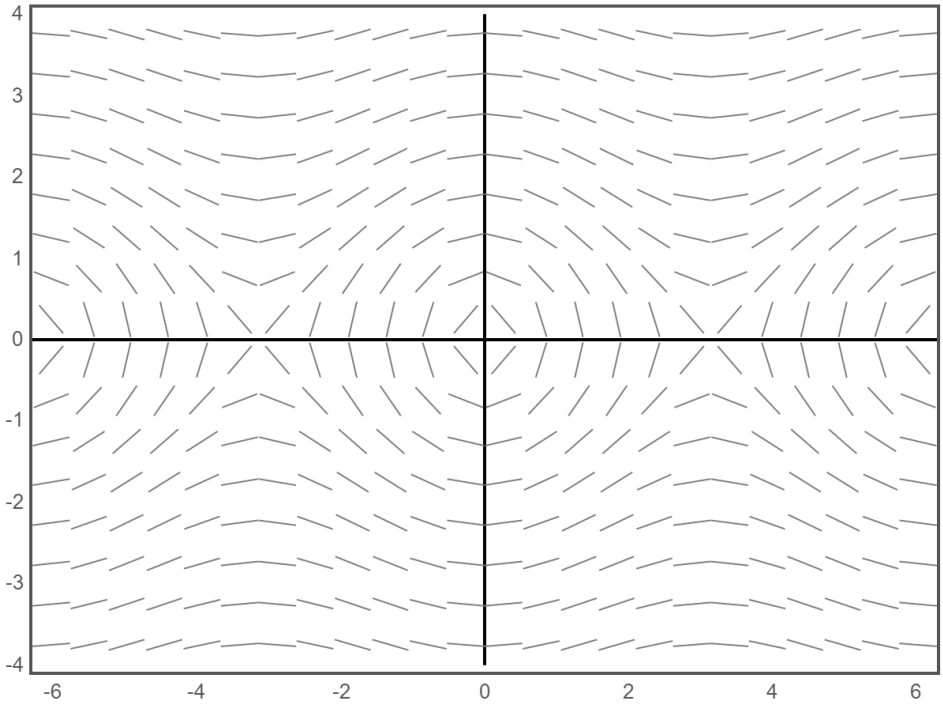
MTH 291, Practice Exam #1, Fall 2018 Name _____

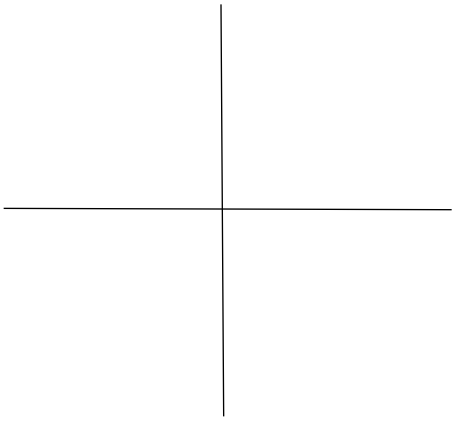
Instructions: Show all work. Answers without work required to obtain the solution will not receive full credit. Some questions may contain multiple parts: be sure to answer all of them. Give exact answers unless specifically asked to estimate.

1. Verify that $y(x) = \ln(x + C)$ is a solution to the differential equation $e^y y' = 1, y(0) = 0$.

2. Shown below is the slope field for an undamped pendulum. Plot at least three sample trajectories (integral solutions) with different behaviors. Track the path forward and backward in time (so that the path begins and ends on the edges of the graph.



3. Use the Existence and Uniqueness Theorem to determine the regions where a solution to the ODE $y^2(xy' + y)\sqrt{1 + x^4} = x$ is guaranteed to exist. Sketch the region in the plane. Be sure to check all conditions and show your work.



4. Solve $x^2y' = 1 - x^2 + y^2 - x^2y^2$ by separation of variables. [Hint: Factor by grouping.]

5. Classify each differential equation as i) linear or nonlinear, ii) state its order.
   a. $x^2y'' + 5xy' + 4y = 0$

b. $\frac{du}{dx} + \frac{d^2u}{dx^2} = xu$

c. $\frac{d^3y}{dt^3} + 2\cos t \frac{d^2y}{dt^2} + \left(\frac{dy}{dt}\right)^2 = e^t$

d. $u^{(5)} + u''' = \ln(u)$

6. Determine if the differential equation is homogeneous. Explain your reasoning.
   a. $x^2y'' + 2xy' - 6y = 0$

   b. $x^2y' + 2xy = 5x^4$

7. Determine the solution method for each of the following equations. Do not solve. Choose from: i) separable, ii) linear, iii) exact, iv) Bernoulli, v) homogeneous, vi) none of these.
   a. $x^2y' + 2xy = 5y^4$

   b. $y' + 2xy^2 = 0$

   c. $\frac{dy}{dx} = x^2 - y$

   d. $\frac{dy}{dx} = \frac{(x-1)y^5}{x^2(2y^3-y)}$

   e. $(1 + ye^{xy})dx + (2y + xe^{xy})dy = 0$

   f. $(x^2 - y^2)y' = 2xy$

8. A 400 gal tank initially contains 100 gal of brine containing 50 lbs of salt. Brine containing 1 lbs of salt per gal enters the tank at a rate of 5 gal/s, and the well-mixed brine flows out of the tank at the rate of 3 gal/s.
   a. Find an equation for the amount of salt in the tank at time *t*.
   b. How much salt will the tank contain when the tank is completely full?
   c. What is the concentration in the salt at time t?
   d. What is the limiting concentration of salt in the tank as $t \to \infty$?
   e. At what time *t* is the concentration equal to 90% of its limiting value?

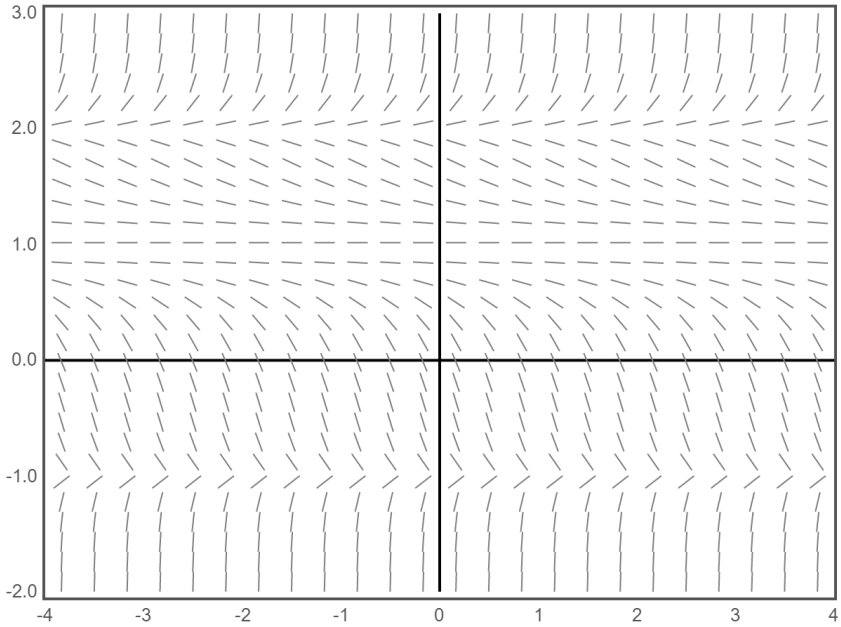
9. Use the method of integrating factors to find the particular solution for $xy' - 3y = 2x^4e^x$.

10. Solve the Bernoulli equation $x^2y' + 2xy = 5y^4$ as a linear equation.

11. Verify that the equation $(2x + y^2)dx + (2xy)dy = 0$ is exact. Then find the general solution.

12. Solve the homogeneous equation $(x + 2y)y' = y$, and state its order.

13. Consider the slope field shown below. If the equilibria are assumed to be integer values, write a differential equation that can produce the field. Characterize each equilibrium as stable, unstable or semistable.



14. Describe the conditions in a population model needed to label an equilibrium as a carrying capacity.

15. Use Euler's Method for 5 steps to approximate the solution of $xy' = y^2, y(1) = 1$ at the point $y(2)$.

16. Estimate the solution of the ODE $\frac{dy}{dx} = y\cos x\,, y(0) = 1$ using $\Delta t = 0.1$ using two complete steps of Runge-Kutta.

17. Use an integrating factor to solve $e^x dx + (e^x \cot y + 2y \csc y)dy = 0$.

18. Find the value of k that will make the equation $(3x^2 - kxy + 2)dx + (6y^2 - x^2 + 3)dy = 0$ exact.

19. Suppose that the temperature of a cup of coffee obeys Newton's law of cooling. If the coffee has a temperature of 200° when freshly poured, and 1 minute later cooled to 180° in a room at 72°, determine when the coffee reaches a temperature of 120° (in minutes).

20. Xanax takes about 11.2 hours after peak levels to reach 50% of of that level in the blood stream. If a drug test can read levels at 5% of peak levels for a single dose, how long will it take to clear the system according to the test? Write a differential equation to solve the problem.