MTH 174, Practice Exam #3, Summer 2018 Name ____

Instructions: Show all work. Give exact answers unless specifically asked to round. All complex numbers should be stated in standard form, and all complex fractions should be simplified. If you do not show work, problems will be graded as "all or nothing" for the answer only; partial credit will not be possible and any credit awarded for the work will not be available.

1. Find the limit of the **sequence** $a_n = \cos\frac{2}{n}$.

2. Simplify $\frac{(2n+2)!}{(2n)!}$.

3. Determine if the **sequence** $a_n = \frac{3n}{n+2}$ is bounded and monotonic.

4. Find the sum of series.
   a. $\sum_{n=0}^{\infty} 5\left(\frac{2}{3}\right)^n$

   b. $\sum_{n=1}^{\infty} \frac{4}{n(n+2)}$

5. Consider the telescoping series $\sum_{k=1}^{\infty}\left(\frac{1}{k} - \frac{1}{k+2}\right)$.
   a. Find a formula for the nth partial sum S_n.

   b. Find the sum of the series.

6. Use the integral test to determine if the series $\sum_{n=1}^{\infty} \frac{\ln n}{n^2}$ converges or diverges.

7. Determine if the series converges. Use the indicated test. Be sure to state all the required elements of the test, including any assumptions (and prove that they apply), in your work.
   a. $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^3+1}}$, direct comparison

   b. $\sum_{n=1}^{\infty} \frac{n}{(n+1)2^{n-1}}$, ratio test

   c. $\sum_{n=2}^{\infty} \frac{1}{5^n+1}$, limit comparison

   d. $\sum_{n=0}^{\infty} \frac{6^n}{(n+1)^n}$, root test

   e. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}\sqrt{n}}{n+2}$, alternating series test

   f. $\sum_{n=1}^{\infty} \frac{ne^{\sqrt{n}}}{n^2+1{,}000{,}000}$, test for divergence

8. Determine the convergence or divergence of the series. State the test used and explain your reasoning for selecting that test.
   a. $\sum_{n=0}^{\infty} \frac{2^n}{n!}$

   b. $\sum_{n=2}^{\infty} \frac{n}{(\ln n)^n}$

   c. $\sum_{n=1}^{\infty} \left(\frac{1}{n^2} - \frac{n}{3^n+1}\right)$

9. Determine if the series $\sum_{n=0}^{\infty} \frac{\cos n\pi}{n+1}$ converges conditionally or absolutely.

10. Consider the infinite series $\sum_{n=1}^{\infty} \frac{4}{n(n+2)}$.
    a. Use the ratio test to determine the convergence of the series. Explain why the test is inconclusive.
    b. Show that the series is decreasing.
    c. Use the integral test to determine if the series converges or diverges.

11. Determine the interval and radius of convergence of x for the series below. Be sure to check the endpoints.
    a. $\sum_{n=1}^{\infty} \frac{(-1)^n (x+1)^n}{n}$

    b. $\sum_{n=0}^{\infty} n! \left(\frac{x}{2}\right)^n$

12. Write the functions as a power series centered at c, by applying the geometric series formula $\sum_{n=0}^{\infty} ax^n = \frac{a}{1-x}$.
    a. $f(x) = \frac{4}{3x+2}, c = 3$

    b. $f(x) = \frac{4x}{(x+1)^2}$

13. Find a Taylor polynomial for the function centered at c for the given number of terms.
    a. $f(x) = \ln(x), n = 6, c = 1$

    b. $f(x) = e^{3x}, n = 4, c = 0$

14. Find the Taylor polynomial with $n = 3$ for the function $f(x) = e^{-x}$, centered at $c = 0$. Use your Taylor polynomial to estimate the value of the function at $x = 0.1$.