MTH 174, Practice Exam #2, Summer 2018 Name ____

Instructions: Show all work. Give exact answers unless specifically asked to round. All complex numbers should be stated in standard form, and all complex fractions should be simplified. If you do not show work, problems will be graded as "all or nothing" for the answer only; partial credit will not be possible and any credit awarded for the work will not be available.

1. Integrate the following integrals using the table of integrals formula provided. Substitution may be needed.
   a. $\int t^3 e^{4t}\, dt$ using $\int x^3 e^{ax} dx = \frac{a^3x^3 - 3a^2x^2 + 6ax - 6}{a^4} e^{ax} + C$.

   b. $\int x^2\sqrt{2+9x^2}dx$ using
   $\int x^2\sqrt{x^2 \pm a^2}dx = \frac{x}{8}\left(2x^2 \pm a^2\right)\sqrt{x^2 \pm a^2} - \frac{a^4}{8}\ln\left|x + \sqrt{x^2 \pm a^2}\right| + C$.

   c. $\int \frac{e^x}{1-\tan e^x} dx$ using $\int \frac{1}{1 \pm \tan x} dx = \frac{1}{2}\left[x \pm \ln\left|\cos x \pm \sin x\right|\right] + C$.

2. Integrate $\int \frac{\cos\left(\frac{1}{\theta}\right)}{\theta^2} d\theta$.

3. Integrate using integration by parts.
   a. $\int \frac{2x+1}{\sqrt{x+4}} dx$

   b. $\int \arctan x\, dx$

   c. $\int e^{4x} \cos 2x\, dx$

   d. $\int x^3 \sin x\, dx$

   e. $\int x^3 e^{x^2}\, dx$

4. Integrate.
   a. $\int \cos^4 x\, dx$

   b. $\int \tan^5 2x\, dx$

5. Use trig substitution to integrate
   a. $\int \frac{1}{(1-t^2)^{5/2}} dt$

   b. $\int \frac{1}{\sqrt{9+2x^2}} dx$

   c. $\int \frac{x^2}{\sqrt{x^2-4}} dx$

6. Rewrite $\int \frac{x^4+3x^2+x}{(x-1)(x+2)^2(x^2+4)}dx$ with partial fractions. Do not solve for the constants or integrate.

7. Integrate using partial fractions.
   a. $\int \frac{2}{9x^2-1}dx$

   b. $\int \frac{x-1}{x(x+2)(x+1)}dx$

8. Evaluate the improper integrals or show that it diverges. Explain why they are improper.
   a. $\int_0^{\infty} \frac{1}{e^x+e^{-x}}dx$

   b. $\int_1^{\infty} \frac{1}{x \ln x}dx$

9. Explain how to determine if an integral is improper, how to evaluate it and determine convergence or divergence.

10. Explain the meaning of the integral $\frac{1}{2}\int_0^2 [(-x+4)^2 - (x)^2]dx$ that we've encountered in a previous section of the course.

11. Explain the meaning of the integral $\int_1^9 \sqrt{1+\frac{1}{(\sqrt{x})}}dx$ that we've encountered in a previous section of the course.

12. Use the definition of hyperbolic functions to answer the following questions.
    a. Prove that $\sinh 2x = 2\sinh x \cosh x$.

    b. Prove that $\frac{d}{dx}[\tanh x] = \operatorname{sech}^2 x$.

13. Find the derivative of the following functions.
    a. $f(x) = e^{\cosh x^2}$

    b. $g(x) = \ln(\tanh x - \operatorname{sech} x)$

    c. $h(x) = \sinh x \cosh^2 x$

14. Integrate.
    a. $\int \operatorname{csch}^2 x\, dx$

    b. $\int \sinh x \cosh^2 x\, dx$

    c. $\int \frac{\operatorname{sech}^2 x}{1+\tanh^2 x}dx$